Key

CALCULUS BC
WORKSHEET ON SERIES

Work the following on notebook paper. Use your calculator only on 10(b).

C 1. Which of the following is a term in the Taylor series about $x = 0$ for the function $f(x) = \cos(2x)$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$ $1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!}$ $\frac{16x^4}{24}$

C 2. Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-3)^n}$ converges.

(A) $x = 2$ (B) $-1 \le x < 5$ (C) $-1 < x \le 5$ (D) $-1 < x < 5$ (E) All real numbers

$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)(-3)^{n+1}} \cdot \frac{n(-3)^n}{(x-2)^n} \right|$ $\lim_{n\to\infty} \left| \frac{(x-2)}{-3} \cdot \frac{n}{n+1} \right|$

B 3. Let $f(x) = \sum_{n=1}^{\infty} (\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$.

(A) $-\frac{1}{7}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{8}{9}$ (E) The series diverges.

$\left(\cos\frac{2\pi}{3}\right)^{3n}$ $\left(\left(\cos\frac{2\pi}{3}\right)^3\right)^n = \left(-\frac{1}{8}\right)^n$ $\frac{-\frac{1}{8}}{1+\frac{1}{8}} = -\frac{1}{8} \cdot \frac{8}{9}$

$-1 < \frac{x-2}{-3} < 1$ $3 > x-2 > -3$ $5 > x > -1$

$x = -1$: $\frac{(-3)^n}{n(-3)^n} = \frac{1}{n}$ di

$x = 5$: $\frac{3^n}{n(-1)^n(3)^n} = \frac{1}{n(-1)^n}$ Con

E 4. Find the sum of the geometric series $\frac{9}{8} - \frac{3}{4} + \frac{1}{2} - \frac{1}{3} + \ldots$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

$r = -2/3$ $-\frac{3}{4} \cdot \frac{8}{9}$ $-\frac{2}{3}$ $\frac{\frac{9}{8}}{1+\left(+\frac{2}{3}\right)} = \frac{9}{8} \cdot \frac{3}{5}$

D 5. The series $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \ldots + \frac{x^{2n+1}}{n!} + \ldots$ is the Maclaurin series for

(A) $x\ln(1+x^2)$ (B) $x\ln(1-x^2)$ (C) e^{x^2} (D) xe^{x^2} (E) $x^2e^{x^2}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

D 6. The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

$1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}$ $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!}$

B 7. The Taylor polynomial of order 3 at $x = 0$ for $f(x) = \sqrt{1+x}$ is

(A) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ (B) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ (C) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$

(D) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$ (E) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ $1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!}$ $1 + \frac{\frac{1}{2}(x-1)}{1!} - \frac{\frac{1}{4}(x-1)^2}{2!}$ $\frac{8}{6} = \frac{4}{3}$

$f(x) = (1+x)^{1/2}$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $f''(0) = -\frac{1}{4}$

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