

The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.

- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

$$(a) \quad W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285 \text{ or } -0.286$$

When $v = 20$ mph, the wind chill is decreasing at $0.286^{\circ}\text{F}/\text{mph}$.

$$(b) \quad \text{The average rate of change of } W \text{ over the interval } 5 \leq v \leq 60 \text{ is } \frac{W(60) - W(5)}{60 - 5} = -0.253 \text{ or } -0.254.$$

$$W'(v) = \frac{W(60) - W(5)}{60 - 5} \text{ when } v = 23.011.$$

$$(c) \quad \left. \frac{dW}{dt} \right|_{t=3} = \left(\frac{dW}{dv} \cdot \frac{dv}{dt} \right) \Big|_{t=3} = W'(35) \cdot 5 = -0.892^{\circ}\text{F}/\text{hr}$$

OR

$$W = 55.6 - 22.1(20 + 5t)^{0.16}$$

$$\left. \frac{dW}{dt} \right|_{t=3} = -0.892^{\circ}\text{F}/\text{hr}$$

Units of $^{\circ}\text{F}/\text{mph}$ in (a) and $^{\circ}\text{F}/\text{hr}$ in (c)

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$$

$$3 : \begin{cases} 1 : \frac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \text{or} \\ \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$$

1 : units in (a) and (c)