AP Calculus BC

KEY

INTEGRATION BY PARTS
Review & Renew

1. $\int x\ln x\,dx$ $\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$	11. $\int \cos x \ln(\sin x)dx$ $\sin x\ln(\sin x) - \sin x + C$
2. $\int x\sec^2 x\,dx$ $x\tan x + \ln\lvert\cos x\rvert + C$	12. $\int \frac{\ln x}{x^2}dx$ $-\frac{\ln x}{x} - \frac{1}{x} + C$
3. $\int x\cos 5x\,dx$ $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$	13. $\int (x\sqrt{x+3})dx$ $\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$
4. $\int (\ln x)^2 dx$ $x(\ln x)^2 - 2(x\ln x - x) + C$ $x(\ln x)^2 - 2x\ln x + 2x + C$	14. $\int \sin^2 x\,dx$ $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ $\frac{1}{2}x - \frac{1}{4}(2\sin x\cos x) + C$
5. $\int xe^{-x}dx$ $-xe^{-x} - e^{-x} + C$	15. $\int \frac{x}{e^x}dx$ $-xe^{-x} - e^{-x} + C$
6. $\int \sin^{-1}x\,dx$ $x\sin^{-1}x + (1-x^2)^{1/2} + C$	16. $\int e^x \sin x\,dx$ $\frac{1}{2}(e^x\sin x - e^x\cos x) + C$
7. $\int x\sin(3x)dx$ $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin(3x) + C$	17. $\int x^2\sin x\,dx$ $-x^2\cos x + 2x\sin x + 2\cos x + C$
8. $\int \ln(2x+1)dx$ $x\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln(2x+1) + C$	18. $\int \ln x\,dx$ $x\ln x - x + C$
9. $\int \cos(\ln x)dx$ $\frac{1}{2}(x\cos(\ln x) + x\sin(\ln x)) + C$	19. $\int xe^{5x}dx$ $\frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$
10. $\int x^3e^x dx$ $e^x(x^3 - 3x^2 + 6x - 6) + C$	20. $\int x^2e^{5x}dx$

8) $\frac{1}{2}[(2x+1)\ln(2x+1) - (2x+1)] + C$ $\frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C$

From: Calculus, *Stewart, 5th ed.*
and
Calculus, *Hughes-Hallett, Gleason, McCallum, et al., 3rd ed.*

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Integration by Parts WS: ILATE $-c \; {}^{S}{}_{-S} \; c$

1) $\int x \ln x \, dx$ $\quad u = \ln x \quad dv = x\,dx$

$\quad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$\ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \circ \frac{1}{x}\,dx = \frac{x^2}{2} \cdot \ln x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2} \cdot \frac{x^2}{2} + C = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

2) $\int x \sec^2 x \, dx$ $\quad u = x \quad dv = \sec^2 x\,dx$

$\quad du = dx \quad v = \tan x$

$x\tan x - \int \tan x\,dx = x\tan x + \ln|\cos x| + C$

$\int \frac{\sin x}{\cos x}dx$ $\quad u = \cos x$

$\quad du = -\sin x\,dx$

$\quad -du = \sin x\,dx$

$-\int \frac{1}{u}du = -\ln|\cos x| + C$

3) $\int x \cos 5x \, dx$ $\quad u = x \quad dv = \cos 5x\,dx$

$\quad du = dx \quad v = \frac{1}{5}\sin 5x$

$\frac{1}{5}x \cdot \sin 5x - \frac{1}{5}\int \sin 5x\,dx$ $\quad u = 5x$

$\quad du = 5dx$

$-\frac{1}{5} \cdot \frac{1}{5}\int \sin u\,du \quad \frac{1}{5}du = dx$

$\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$

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4) $\int (\ln x)^2 dx = \int (\ln x \cdot \ln x)\, dx$

$u = \ln x \quad dv = \ln x\, dx$

$du = \frac{1}{x} dx \quad v = x\ln x - x$

$\ln x (x\ln x - x) - \int (x\ln x - x)(\frac{1}{x}) dx$

$x(\ln x)^2 - x\ln x - \int (\ln x - 1)\, dx \qquad \int \ln x \cdot 1 dx$

$x(\ln x)^2 - x\ln x - (\underbrace{x\ln x - x - x}_{-2x}) + C$

$x(\ln x)^2 - x\ln x - x\ln x + 2x + C$

$x(\ln x)^2 - 2x\ln x + 2x + C$

5) $\int x e^{-x} dx$

	u	dv
$+$	x	e^{-x}
$-$	1	$-e^{-x}$
	0	e^{-x}

$-xe^{-x} - e^{-x} + C$

6) $\int \sin^{-1} x \cdot 1 dx$

$u = \sin^{-1} x \quad dv = 1\, dx$

$du = \frac{1}{\sqrt{1-x^2}} \quad v = x$

$x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$

$u = 1 - x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$x\sin^{-1}x + \frac{1}{2}\int u^{-1/2} du \to 2u^{1/2}$

$x\sin^{-1}x + \frac{1}{2}\cdot 2(1-x^2)^{1/2} + C$

$x\sin^{-1}x + (1-x^2)^{1/2} + C$

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7) $\int x \sin(3x)\,dx$

	u	dv
$+$	x	$\sin(3x)$
$-$	1	$-\frac{1}{3}\cos(3x)$
	0	$-\frac{1}{9}\sin(3x)$

$-\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C$

8) $\int \ln(2x+1)\,dx$

$u = \ln(2x+1)$ $\quad dv = 1\,dx$

$du = \frac{1}{2x+1}\cdot 2\,dx$ $\quad v = x$

$x\ln(2x+1) - 2\int x \cdot \frac{1}{2x+1}\,dx$

$u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{u-1}{2} = x$

$x\ln(2x+1) - 2\cdot\frac{1}{2}\int\left(\frac{u-1}{2}\right)\left(\frac{1}{u}\right)du$

$x\ln(2x+1) - \frac{1}{2}\int \frac{u-1}{u}\,du$

$x\ln(2x+1) - \frac{1}{2}\left(\int 1\,du - \int \frac{1}{u}\,du\right)$

$x\ln(2x+1) - \frac{1}{2}\left(u - \ln|u|\right) + C$

$x\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln(2x+1) + C$

OR

$u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int \ln u\,du$

$\frac{1}{2}(u\ln u - u) + C$

$\frac{1}{2}\left[(2x+1)\ln(2x+1) - (2x+1)\right] + C$

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9) $\int \cos(\ln x)\,dx$

$u = \ln x \qquad x = e^u$

$du = \frac{1}{x}dx$

$x\,du = dx$

$e^u du = dx$

$\int \cos u \cdot e^u du$

$a = \cos u \qquad dv = e^u du$

$da = -\sin u\,du \qquad v = e^u$

$\cos u \cdot e^u - \int e^u \cdot -\sin u\,du$

$e^u \cos u + \int e^u \sin u\,du$

$a = \sin u \quad dv = e^u du$

$da = \cos u\,du \quad v = e^u$

$e^u \cos u + \left(\sin u \cdot e^u - \int e^u \cdot \cos u\,du\right)$

$\int \cos u \cdot e^u du = e^u \cos u + e^u \sin u - \int e^u \cos u\,du$

$2\int e^u \cos u\,du = e^u \cos u + e^u \sin u$

$\int e^u \cos u\,du = \frac{1}{2}\left(e^u \cos u + e^u \sin u\right) + C$

$\frac{1}{2}\left(\cancel{e}^{\cancel{\ln} x} \cos(\ln x) + \cancel{e}^{\cancel{\ln} x} \sin(\ln x)\right) + C$

$\frac{1}{2}\left(x\cos(\ln x) + x\sin(\ln x)\right) + C$

OR

$u = \cos(\ln x) \qquad v = x$

$du = -\frac{\sin(\ln x)}{x} \qquad dv = 1\,dx$

$x\cos(\ln x) + \int \sin(\ln x)\,dx$

$u = \sin(\ln x) \qquad v = x$

$du = \frac{\cos(\ln x)}{x}dx \qquad dv = 1dx$

$x\sin(\ln x) - \int \cos(\ln x)\,dx$

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10) $\int x^3 e^x \, dx$

	u	dv
+	x^3	e^x
−	$3x^2$	e^x
+	$6x$	e^x
−	6	e^x
	0	e^x

$$e^x(x^3 - 3x^2 + 6x - 6) + C$$

11) $\int \cos x \cdot \ln(\sin x)\, dx$

$u = \ln(\sin x)$ $\quad dv = \cos x\, dx$

$du = \frac{1}{\sin x} \cdot \cos x\, dx$ $\quad V = \sin x$

$$\ln(\sin x) \cdot \sin x - \int \cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} \cdot \cos x \, dx$$

$$\sin x \ln(\sin x) - \int \cos x \, dx$$

$$\sin x \ln(\sin x) - \sin x + C$$

12) $\int \frac{\ln x}{x^2} dx$

$u = \ln x$ $\quad dv = x^{-2} dx$

$du = \frac{1}{x} dx$ $\quad V = -x^{-1}$

$$\int (x^{-2} \cdot \ln x)\, dx$$

$$\ln x \cdot \frac{1}{x^2} dx$$

$$\ln x \cdot x^{-2} dx$$

$$-\frac{\ln x}{x} + \int \frac{1}{x} \cdot \frac{1}{x} dx$$

x^{-2}

$$-\frac{\ln x}{x} - \frac{1}{x} + C$$

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13) $\int \left(x\sqrt{x+3}\right) dx$ $u = x+3$ $x = u-3$

$du = dx$

$\int (u-3)\cdot u^{1/2}\, du$

$\int u^{3/2} - 3u^{1/2}\, du$

$\int u^{3/2} du - 3\int u^{1/2} du$

$\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C$

$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

14) $\int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2} dx$

$\int \frac{1}{2} dx - \frac{1}{2}\int \cos 2x\, dx$ $u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos u\, du$

$\frac{1}{2}x - \frac{1}{4}\sin 2x + C$

15) $\int \frac{x}{e^x} dx = \int x\cdot e^{-x} dx$

	u	dv
$+$	x	e^{-x}
$-$	1	$-e^{-x}$
	0	e^{-x}

$-xe^{-x} - e^{-x} + C$

$\int e^{-x} dx$

$-e^{-x}$

16) $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x dx$
$du = \cos x \, dx \qquad v = e^x$

$e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$
$du = -\sin x \, dx \qquad v = e^x$

$e^x \sin x - \left(e^x \cos x + \int e^x \sin x \, dx\right)$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$

17) $\int x^2 \sin x \, dx$

	u	dv
$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$
	0	$\cos x$

$-x^2 \cos x + 2x \sin x + 2\cos x + C$

18) $\int \ln x \, dx$

$u = \ln x \qquad dv = 1\, dx$
$du = \frac{1}{x} dx \qquad v = x$

$x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx = x \ln x - x + C$

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19) $\int x e^{5x}\,dx$

	u	dv
+	x	e^{5x}
−	1	$\frac{1}{5}e^{5x}$
	0	$\frac{1}{25}e^{5x}$

$\frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$

20) $\int x^2 e^{5x}\,dx$

	u	dv
+	x^2	e^{5x}
−	$2x$	$\frac{1}{5}e^{5x}$
+	2	$\frac{1}{25}e^{5x}$
	0	$\frac{1}{125}e^{5x}$

$\frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C$

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