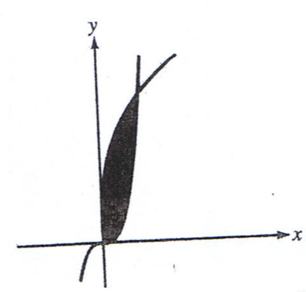
Review of Area & Volume
AP Calculus AB

Name: ______________________
Date: ______________________

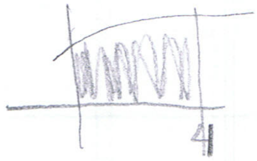
1. Find the volume of the solid formed by rotating about the x-axis the region enclosed by the graph of $y = \sqrt{x} + 1$, the x-axis, the y-axis, and the line x = 4.
2. Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis, and the line y = 3 about the y-axis (calculator)
3. Find the area enclosed by the curve bounded by $f(x) = 5\sqrt{x}$, $g(x) = 4x - 6$ and the y-axis.
4. Find the area enclosed by the curve bounded by f(x) = sinx, g(x) = cos x, y-axis, [0, π/4]
5. Find the area enclosed by the curve bounded by $f(x) = \sqrt{2 - x}$, $g(x) = x^3$, y-axis (calculator)
6. Find the area enclosed by the curve bounded by $y = 3x^3 and\ x = 3y^2 - 5$ (calculator)
7. Find the volume generated when $y = 15 - 2x - x^2$ is rotated about the x-axis on the interval [-5,3] (calculator)
8. The region bounded by the graphs $y = e^x, y = 1, and\ x = -1$ is rotated about the x-axis. Find the volume of the resulting solid.
9. Find the volume generated when $f(x) = x^2 + 6\ and\ g(x) = 5x$ is enclosed by the y-axis and revolved about the x-axis
10. Find the area between the curves $x = e^y$, $x = y^2 - 2$ and the lines y = -1 and y = 1
11. The region enclosed by the curves y = x and $y = x^2$.
    a. Find the volume of the solid if it is rotated by the x-axis.
    b. Find the volume if rotated about the line y = 2
    c. Find the volume if rotated about the line x = -1
12. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.
13. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, y = 8 and x = 0 about the y-axis.
14. Find the volume of the solid obtained by rotating the region bounded by y = 3x, y = 2x, and y = 3 about the y-axis.
15. Find the volume of the solid obtained by rotating the region under the graph $f(x) = 9 - x^2$ from [0, 3] about the vertical axis x = -2.
16. Suppose that a population grows according to the logistic function $\frac{dP}{dt} = 0.05P - 0.0005P^2$
    a. If the initial population is 20, solve for P(t)
    b. What is the carrying capacity
    c. What is the value of k
    d. How long will it take the population to reach a population of 45
    e. What will be the population when t = 5?
    f. What is the limit t→∞ P(t)
    g. When is the population growing the fastest?



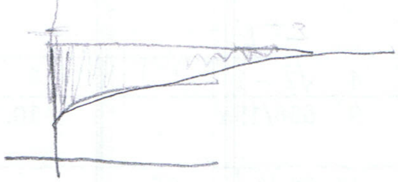
Calculator Inactive

A region is enclosed by the graphs $y = x^3$ and $y = 4\sqrt[3]{4x}$.

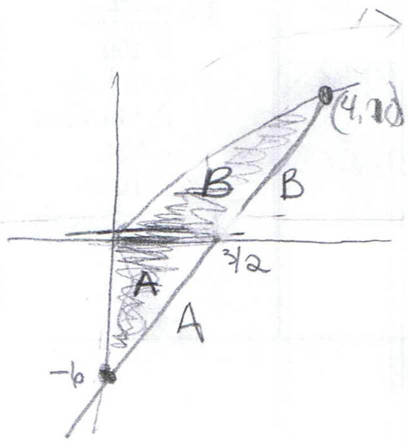
a) Find the coordinates of the point in the first quadrant where the two curves intersect
b) Use an integral with respect to x to find the area of the shaded region.
c) Write an integral with respect to y that could be used to confirm your answer to part (b).
d) Without using absolute values, write an integral expression that gives the volume of the solid generated by revolving the shaded region about the line x = -1. Do not evaluate.

1. $V =$ 

$V = \pi \int_0^4 (\sqrt{x}+1)^2\,dx = \frac{68\pi}{3}$

2. $V =$ 

$V = \pi \int_1^3 ((y-1)^2)^2 = 6.4\pi \approx 20.106$

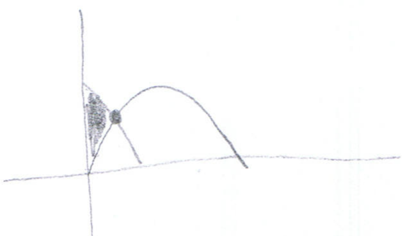
3. 

$x = \frac{y^2}{25}$ $\quad x = \frac{y+6}{4}$

A $\quad$ B

$\int_{-6}^{0} \frac{y+6}{4} + \int_0^{10} \frac{y+6}{4} - \frac{y^2}{25}$

$\frac{9}{2} + \frac{85}{6} = \frac{56}{3}$

4. 

$\int_0^{\pi/4} (\cos x - \sin x)\,dx = \sqrt{2}-1$

5. 

$\int_0^1 \sqrt{2-x} - x^3 = .969$

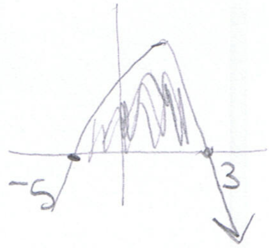
$x = \sqrt[3]{\frac{y}{3}}$ $\quad y = -1.192$

$x = (3y^2-5)$ $\quad y = 1.387$

6. 

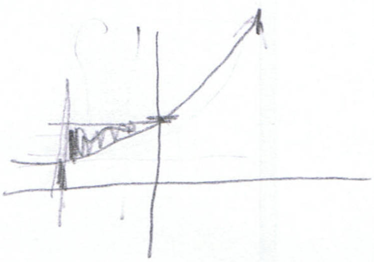
$\int_{-1.192}^{1.387} \left(\sqrt[3]{y/3} - (3y^2-5)\right)dy = 8.680$

7.



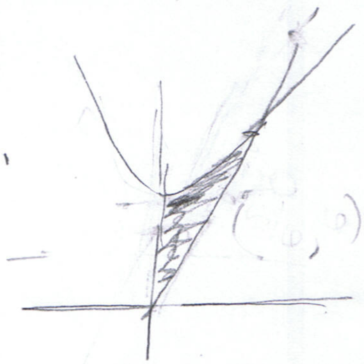
$$\pi\int_{-5}^{3}(15-2x-x^2)^2 = 1092.2667\pi \approx 3431.457$$

8.



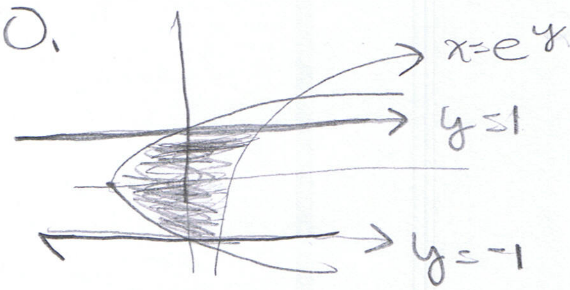
$$\pi\int_{-1}^{0}(1)^2-(e^x)^2 = \frac{\pi(1+e^{-2})}{2}$$

9.



$$\pi\int_{0}^{2}(x^2+6)^2-(5x)^2 = \frac{656}{15}\pi$$

10.



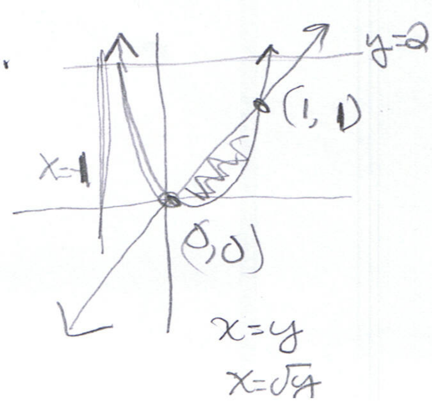
$$\int_{-1}^{1}\left(e^y-(y^2-2)\right)dy$$

$$= e^y-\frac{y^3}{3}+2y\Big|_{-1}^{1}$$

$$e^1-\frac{1}{3}+2-\left(e^{-1}+\tfrac{1}{3}-2\right)$$

$$\boxed{e-\frac{1}{e}+\frac{10}{3}}$$

11.



a) $\pi\int_0^1 (x)^2-(x^2)^2\,dx = \frac{2\pi}{15}$

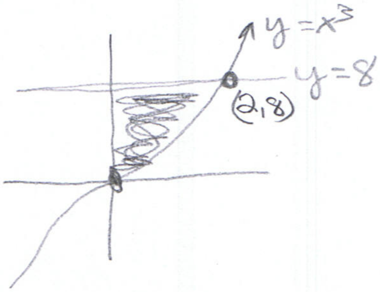
b) $\pi\int_0^1 (2-x^2)^2-(2-x)^2 = \frac{8\pi}{15}$

c) $\pi\int^1 (\sqrt{y}-1)^2-(y-1)^2 = \pi/2$

12.

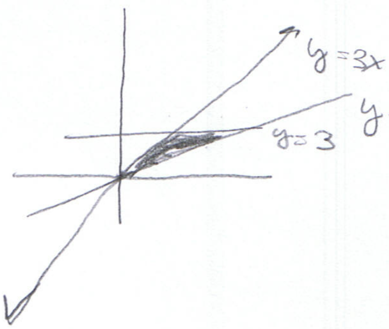
$$V = \pi\int_0^1 (\sqrt{x})^2\,dx = \pi/2$$

13.



$$V = \pi\int_0^8 (\sqrt[3]{y})^2\,dy = \frac{96}{5}\pi$$

14.

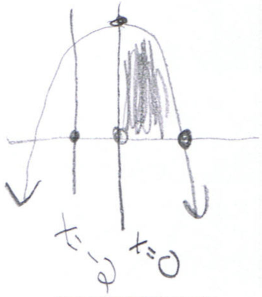


$x = y/3$

$x = y/2$

$$V = \pi\int_0^3 \left[\left(\frac{y}{2}\right)^2 - \left(\frac{y}{3}\right)^2\right] dy = \frac{5}{4}\pi$$

15.



$x = \sqrt{9-y}$

$$\pi\int_0^9 (\sqrt{9-y} - -2)^2 - (0-(-2))^2$$

$$\pi\int_0^9 \left[(\sqrt{9-y}+2)^2 - 4\right] dy = \frac{225}{2}\pi$$

16. a) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{100}\right)$

$M = 100 \quad K = .05$

$y = \frac{100}{1+Ae^{-.05t}}$

$20 = \frac{100}{1+Ae^0}$

$1 + A = \frac{100}{20}$

$A = 4$

a) $y = \frac{100}{1+4e^{-.05t}}$

b) 100

c) $K = .05$

d) $45 = \frac{100}{1+4e^{-.05t}}$ $\quad t = 24$

e) $\frac{100}{1+4e^{(.05\cdot 5)}} = 24$

f) 100

g) $50 = \frac{100}{1+4e^{-.05t}}$ $\quad t \approx 50$ yea