$\langle x(t), y(t) \rangle$ position vector

$\langle x'(t), y'(t) \rangle$ velocity vector

$\langle x''(t), y''(t) \rangle$ acceleration vector

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ = speed of particle
or
magnitude (length) of
velocity vector

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ = length of curve (arc length)
or
distance traveled
by particle
from a to b

Vectors and Motion (Notes and HW):

1. A particle moves in the *xy*-plane so that at any time *t*, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$. $s(t) = \langle t^5 - 1, 3t^4 - 2t^3 \rangle$ $v(t) = \langle 5t^4, 12t^3 - 6t^2 \rangle$ $a(t) = \langle 20t^3, 36t^2 - 12t \rangle$ $a(1) = \langle 20, 24 \rangle$	2. A particle moves on the curve $y = \ln x$ so that its *x*-component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point (1, 0). Find the position of the particle at time $t = 1$. x y $x(t) = \int (t+1)\,dt$ $x(t) = \frac{t^2}{2} + t + C$ $1 = 0 + 0 + C$ $C = 1$ $x(t) = \frac{t^2}{2} + t + 1$ $x(1) = \frac{1}{2} + 1 + 1 = \frac{5}{2}$ $y = \ln x \quad y = \ln\frac{5}{2} \quad \left(\frac{5}{2}, \ln\frac{5}{2}\right)$
3. The position of a particle moving in the *xy*-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of *t* is the particle at rest? $(v = 0)$ $x'(t) = 3t^2 - 3t - 18 = 0$ $x'(t) = 3(t^2 - t - 6) = 0$ $3(t-3)(t+2) = 0$ $t = 3, -2$ $y'(t) = 3t^2 - 12t + 9 = 0$ $y'(t) = 3(t^2 - 4t + 3) = 0$ $3(t-3)(t-1) = 0$ $t = 3, 1$ $t = 3$	4. A curve *C* is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of *C* at the point $(8, -4)$. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-5}{3t^2}$ $8 = t^3 \qquad -4 = t^2 - 5t + 2$ $t = 2 \qquad \vdots$ $t = 3, 2$ $\left.\frac{dy}{dx}\right\|_{t=2} = \frac{2(2)-5}{3(2)^2} = \frac{-1}{12}$ $y + 4 = -\frac{1}{12}(x - 8)$

The position of a particle at any time $t \ge 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

(a) Find the magnitude of the velocity vector at time $t = 5$.
(b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.
(c) Find $\frac{dy}{dx}$ as a function of x.

a) $\left.\sqrt{(2t)^2 + (2t^2)^2}\right|_{t=5}$
$\sqrt{2600} = 10\sqrt{26}$

b) $\int_0^5 \sqrt{(2t)^2 + (2t^2)^2}\,dt$
$= \frac{2}{3}(26^{3/2} - 1)$

c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t = \sqrt{x+3}$
$x = t^2 - 3$
$x + 3 = t^2$
$t = \sqrt{x+3}$

6. Point $P(x, y)$ moves in the xy-plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \ge 0$.

(a) Find the coordinates of P in terms of t given that $t = 1$, $x = \ln 2$, and $y = 0$.
(b) Write an equation expressing y in terms of x.
(c) Find the average rate of change of y with respect to x as t varies from 0 to 4.
(d) Find the instantaneous rate of change of y with respect to x when $t = 1$

a) $x(t) = \int \frac{1}{t+1}\,dt = \ln|t+1| + C$
$C = 0$
$x(t) = \ln(t+1)$
$y(t) = \int 2t\,dt = t^2 + D$
$D = -1$
$y(t) = t^2 - 1$
$(x, y) = (\ln(t+1), t^2 - 1)$

b) $x = \ln(t+1)$
$e^x = t + 1$
$t = e^x - 1$
$y = (e^x - 1)^2 - 1$
$y = e^{2x} - 2e^x$

c) $\frac{y(b) - y(a)}{x(b) - x(a)} = \frac{y(4) - y(0)}{x(4) - x(0)} = \frac{16}{\ln 5}$

d) $\left.\frac{dy}{dx}\right|_{t=1} = \frac{2t}{\frac{1}{t+1}} = 4$

You Try:

1. If a particle moves in the xy-plane so that at time t its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.

$\langle -3, 3\pi \rangle$

2. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{-6}{5}$

3. If $x = t^2 - 1$ and $y = e^{t^3}$, find dy/dx.

$\frac{dy}{dx} = \frac{3t \cdot e^{t^3}}{2}$

4. If a particle moves in the xy-plane so that at any time $t > 0$, its position vector is $\langle \ln(t^2 + 5t), 3t^2 \rangle$, find its velocity vector at time $t = 2$.

$\langle \frac{9}{14}, 12 \rangle$

5. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1 + t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, find the position of the particle at $t = 2$.

$(9, 4)$

6. Calculator: A particle moves in the xy-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

$\langle 7.008, -2.228 \rangle$

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You Try:

(1) $x(t) = \langle \sin(3t - \frac{\pi}{2}), 3t^2 \rangle$

$v(t) = \langle 3\cos(3t - \frac{\pi}{2}), 6t \rangle$

$v(\frac{\pi}{2}) = \langle -3, 3\pi \rangle$

(2) $xy = 10$

$x \cdot \frac{dy}{dt} + y\frac{dx}{dt} = 0$

$2(3) + 5(\frac{dx}{dt}) = 0$

$\frac{dx}{dt} = \frac{-6}{5}$

(3) $x = t^2 - 1$ $\quad \frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t} = \frac{3te^{t^3}}{2}$

$y = e^{t^3}$

(4) $x(t) = \langle \ln(t^2+5t), 3t^2 \rangle$

$v(t) = \langle \frac{2t+5}{t^2+5t}, 6t \rangle$

$v(2) = \langle \frac{9}{14}, 12 \rangle$

(5) $v(t) = \langle 1+t, t^3 \rangle$

$x(t) = \int (1+t)dt = t + \frac{t^2}{2} + C = x(t)$

$0 + 0 + C = 5$

$C = 5$

$x(t) = t + \frac{t^2}{2} + 5$

$y(t) = \int t^3 dt = \frac{t^4}{4} + C = y(t)$

$0 + C = 0$

$C = 0$

$y(t) = \frac{t^4}{4}$

$s(t) = \langle t + \frac{t^2}{2} + 5, \frac{t^4}{4} \rangle$

$s(2) = \langle 9, 4 \rangle$

6 $v(t) = \langle 5 + 3\cos t, (8-t)(\sin t) + (1-\cos t)(-1) \rangle$

$25 = 5t + 3\sin t$

$t \approx 5.446$

$v(5.446) \approx \langle 7.008, -2.228 \rangle$