2001 AB6

(a) $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}(6-2x) - 2y^2$
$= 2y^3(6-2x)^2 - 2y^2$

$\left.\frac{d^2y}{dx^2}\right|_{\left(3,\frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$

3 :
- 2 : $\frac{d^2y}{dx^2}$
  $<-2>$ product rule or chain rule error
- 1 : value at $\left(3, \frac{1}{4}\right)$

(b) $\frac{1}{y^2}\,dy = (6-2x)\,dx$

$-\frac{1}{y} = 6x - x^2 + C$

$-4 = 18 - 9 + C = 9 + C$

$C = -13$

$y = \frac{1}{x^2 - 6x + 13}$

6 :
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(3) = \frac{1}{4}$
- 1 : solves for y

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

2000 AB6

(a) $e^{2y}\,dy = 3x^2\,dx$

$\frac{1}{2}e^{2y} = x^3 + C_1$

$e^{2y} = 2x^3 + C$

$y = \frac{1}{2}\ln\left(2x^3 + C\right)$

$\frac{1}{2} = \frac{1}{2}\ln(0 + C);\quad C = e$

$y = \frac{1}{2}\ln(2x^3 + e)$

6 {
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \frac{1}{2}$
- 1 : solves for y

Note: 0/1 if y is not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$x^3 > -\frac{1}{2}e$

$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$

Range: $-\infty < y < \infty$

3 {
- 1 : $2x^3 + e > 0$
- 1 : domain
  Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x

2003 AB1/BC1 Form B

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$

$f(3) = 36 - 27 = 9$

Tangent line at $x = 3$ is

$y = -3(x-3) + 9 = -3x + 18,$

which is the equation of line ℓ.

2 :
- 1 : finds $f'(3)$ and $f(3)$
- 1 : finds equation of tangent line or shows (3,9) is on both the graph of f and line ℓ

(b) $f(x) = 0$ at $x = 4$

The line intersects the x-axis at $x = 6$.

$$\text{Area} = \frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right) dx$$

$$= 7.916 \text{ or } 7.917$$

OR

$$\text{Area} = \int_3^4 \left((18 - 3x) - \left(4x^2 - x^3\right)\right) dx + \frac{1}{2}(2)(18 - 12)$$

$$= 7.916 \text{ or } 7.917$$

4 :
- 2 : integral for non-triangular region
  - 1 : limits
  - 1 : integrand
- 1 : area of triangular region
- 1 : answer

(c) $$\text{Volume} = \pi \int_0^4 \left(4x^2 - x^3\right)^2 dx$$

$$= 156.038\pi \text{ or } 490.208$$

3 :
- 1 : limits and constant
- 1 : integrand
- 1 : answer

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

1: Correct limits in an integral in (a), (b), or (c)

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$

$= 0.442$ or 0.443

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$

$= 0.453\pi$ or 1.423 or 1.424

$3 : \begin{cases} 2 : \text{integrand} \\ \quad <-1> \text{ reversal} \\ \quad <-1> \text{ error with constant} \\ \quad <-1> \text{ omits 1 in one radius} \\ \quad <-2> \text{ other errors} \\ 1 : \text{answer} \end{cases}$

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

$3 : \begin{cases} 2 : \text{integrand} \\ \quad <-1> \text{ incorrect but has } \sqrt{x} - e^{-3x} \text{ as a factor} \\ 1 : \text{answer} \end{cases}$

2002 AB1 Form B

Region R $\frac{x^3}{1+x^2} = 4 - 2x$ at $x = 1.487664 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$ $= 3.214$ or 3.215	2 { 1 : integrand 1 : answer
(b) Volume $= \pi \int_0^A \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right) dx$ $= 31.884$ or 31.885 or 10.149π	3 { 2 : integrand and constant $< -1 >$ each error 1 : answer
(c) Volume $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$ $= 8.997$	3 { 2 : integrand $< -1 >$ each error note: 0/2 if not of the form $k\int_c^d (f(x) - g(x))^2\, dx$ 1 : answer

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

$2\begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2 \right) dx$

$= 7.515\pi$ or 23.609

$4\begin{cases} 1: \text{ limits and constant} \\ 2: \text{ integrand} \\ \quad < -1 > \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \quad k\int_a^b \left(R(x)^2 - r(x)^2 \right) dx \\ 1: \text{ answer} \end{cases}$

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$
$h(0.5) = 2.3418$
$h(1) = 2.718$

The absolute minimum is 2.330.
The absolute maximum is 2.718.

$3\begin{cases} 1: \text{ considers } h'(x) = 0 \\ 1: \text{ identifies critical point} \\ \quad \text{and endpoints as candidates} \\ 1: \text{ answers} \end{cases}$

Note: Errors in computation come off the third point.