(a) \[ \frac{d^2y}{dx^2} = 2y \frac{dy}{dx} (6 - 2x) - 2y^2 \]
\[ = 2y^3 (6 - 2x)^2 - 2y^2 \]
\[ \frac{d^2y}{dx^2} \bigg|_{(3.4)} = 0 - 2 \left( \frac{1}{4} \right)^2 = -\frac{1}{8} \]

(b) \[ \frac{1}{y^2} \ dy = (6 - 2x) \ dx \]
\[ -\frac{1}{y} = 6x - x^2 + C \]
\[ -4 = 18 - 9 + C = 9 + C \]
\[ C = -13 \]
\[ y = \frac{1}{x^2 - 6x + 13} \]

2: \[ \frac{d^2y}{dx^2} \]
-2: product rule or chain rule error
3: 
1: value at \((3, \frac{1}{4})\)

1: separates variables
1: antiderivative of \(dy\) term
5: 
6: 
1: constant of integration
1: uses initial condition \(f(3) = \frac{1}{4}\)
1: solves for \(y\)

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration
Note: 0/6 if no separation of variables
(a) \( e^{2y} \, dy = 3x^2 \, dx \)

\[
\frac{1}{2} e^{2y} = x^3 + C_1
\]

\[
e^{2y} = 2x^3 + C
\]

\[
y = \frac{1}{2} \ln(2x^3 + C)
\]

\[
\frac{1}{2} = \frac{1}{2} \ln(0 + C); \quad C = e
\]

\[
y = \frac{1}{2} \ln(2x^3 + e)
\]

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

1: separates variables
1: antiderivative of \( dy \) term
1: antiderivative of \( dx \) term
1: constant of integration
1: uses initial condition \( f(0) = \frac{1}{2} \)
1: solves for \( y \)

Note: 0/1 if \( y \) is not a logarithmic function of \( x \)

(b) Domain: \( 2x^3 + e > 0 \)

\[
x^3 > -\frac{1}{2} e
\]

\[
x > \left( -\frac{1}{2} e \right)^{1/3} = -\left( \frac{1}{2} e \right)^{1/3}
\]

Range: \( -\infty < y < \infty \)

3: domain

Note: 0/1 if 0 is not in the domain

Note: 0/3 if \( y \) is not a logarithmic function of \( x \)
(a) \[ f(x) = 8x - 3x^2; \quad f'(3) = 24 - 27 = -3 \]
\[ f(3) = 36 - 27 = 9 \]
Tangent line at \( x = 3 \) is
\[ y = -3(x - 3) + 9 = -3x + 18, \]
which is the equation of line \( \ell \).

(b) \[ f(x) = 0 \text{ at } x = 4 \]
The line intersects the \( x \)-axis at \( x = 6 \).
Area \[ = \frac{1}{2}(3)(9) - \int_3^4 (4x^2 - x^3) \, dx \]
\[ = 7.916 \text{ or } 7.917 \]
OR
\[ \text{Area } = \int_3^4 ((18 - 3x) - (4x^2 - x^3)) \, dx \]
\[ + \frac{1}{2}(2)(18 - 12) \]
\[ = 7.916 \text{ or } 7.917 \]

(c) Volume \[ = \pi \int_0^4 (4x^2 - x^3)^2 \, dx \]
\[ = 156.038 \pi \text{ or } 490.208 \]
Point of intersection

\[ e^{-3x} = \sqrt{x} \text{ at } (T, 5) = (0.238734, 0.488604) \]

(a) Area = \( \int_{T}^{1} (\sqrt{x} - e^{-3x}) \, dx \)

= 0.442 or 0.443

(b) Volume = \( \pi \int_{T}^{1} \left( (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \right) \, dx \)

= 0.453 \pi \text{ or } 1.423 \text{ or } 1.424

(c) Length = \( \sqrt{x} - e^{-3x} \)

Height = \( 5(\sqrt{x} - e^{-3x}) \)

Volume = \( \int_{T}^{1} 5(\sqrt{x} - e^{-3x})^2 \, dx = 1.554 \)

1: Correct limits in an integral in (a), (b), or (c)

2: \{ 1 : integrand

1 : answer

2 : integrand

< -1 > reversal

< -1 > error with constant

< -1 > omits 1 in one radius

< -2 > other errors

1 : answer

3 : \{ 2 : integrand

< -1 > incorrect but has \( \sqrt{x} - e^{-3x} \) as a factor

1 : answer
Region $R$
\[ \frac{x^3}{1 + x^2} = 4 - 2x \text{ at } x = 1.487664 = A \]

(a) Area = \[ \int_0^A \left( 4 - 2x - \frac{x^3}{1 + x^2} \right) \, dx \]
\[ = 3.214 \text{ or } 3.215 \]

(b) Volume
\[ = \pi \int_0^A \left( (4 - 2x)^2 - \left( \frac{x^3}{1 + x^2} \right)^2 \right) \, dx \]
\[ = 31.884 \text{ or } 31.885 \text{ or } 10.149\pi \]

(c) Volume = \[ \int_0^A \left( 4 - 2x - \frac{x^3}{1 + x^2} \right)^2 \, dx \]
\[ = 8.997 \]
(a) Area = \( \int_{\frac{1}{2}}^{1} (e^x - \ln x) \, dx \) = 1.222 or 1.223

(b) Volume = \( \pi \int_{\frac{1}{2}}^{1} \left( (4 - \ln x)^2 - (4 - e^x)^2 \right) \, dx \)
   = 7.515\pi or 23.609

(c) \( h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0 \)
   \( x = 0.567143 \)

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

\( h(0.567143) = 2.330 \)
\( h(0.5) = 2.3418 \)
\( h(1) = 2.718 \)

The absolute minimum is 2.330.
The absolute maximum is 2.718.