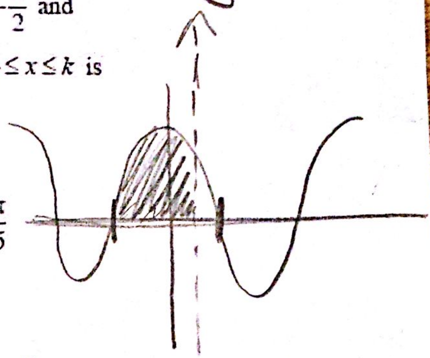
AP CALCULUS AB **Problem Set Unit 6** **Name:** Key

1. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$



(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

$$\int_{-\pi/2}^{K} \cos x\,dx = 3\int_{K}^{\pi/2} \cos x\,dx$$

$$\sin x \Big|_{-\pi/2}^{K} = 3 \sin x \Big|_{K}^{\pi/2}$$

$$\sin K - \sin(-\pi/2) = 3\cdot(\sin\tfrac{\pi}{2} - \sin K)$$

$$\sin K + 1 = 3(1 - \sin K)$$

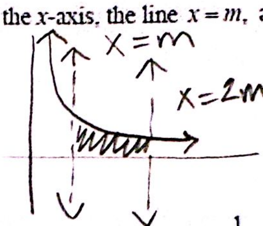
$$\sin K + 1 = 3 - 3\sin K$$

$$4\sin K = 2$$

$$\sin K = \tfrac{1}{2}$$

$$K = \tfrac{\pi}{6}$$

2. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region



$A = \int_{m}^{2m} \frac{1}{x}\,dx$

$\ln|x| \Big|_{m}^{2m}$

$\ln 2m - \ln m$

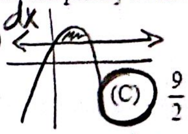
$\ln 2 + \cancel{\ln m} - \cancel{\ln m}$

$\ln 2$

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

3. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

$\int_{-1}^{2} (-x^2 + x + 6 - 4)\,dx$

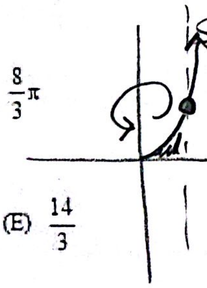


$4 = -x^2 + x + 6$
$0 = -x^2 + x + 2$
$0 = x^2 - x - 2$
$(x-2)(x+1)$
$x = 2, -1$

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

4. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

$x = \sqrt{y}$

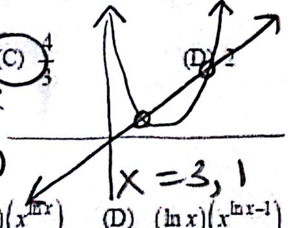


(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

$$\pi\int_{0}^{4} 2^2 - (\sqrt{y})^2\,dy$$

$$\pi\int_{0}^{4} (4 - y)\,dy$$

5. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is



(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

$x^2 - 3x + 3 = x$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = 3, 1$

$$\int_{1}^{3} x - (x^2 - 3x + 3)\,dx$$

6. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

$y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

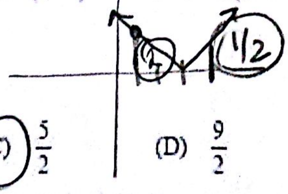
$\ln y = \ln x \cdot \ln x \quad (\ln x)^2$

$\frac{1}{y}\frac{dy}{dx} = 2(\ln x)\cdot\frac{1}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x}\cdot y$

$\frac{2\ln x}{x}\cdot x^{\ln x}$

7. $\int_1^4 |x-3|\,dx =$



2 $\quad 1/2$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

8. The $\lim\limits_{h\to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

$f(x) = \tan 3x$

$f'(x) = \sec^2(3x)\cdot 3$

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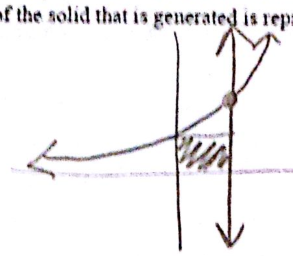
A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

13.177

(A) $2\pi\int_0^1 xe^{2x}\,dx$ (D) $\pi\int_0^e y\ln y\,dy$

(B) $2\pi\int_0^1 e^{2x}\,dx$ (E) $\frac{\pi}{4}\int_0^e \ln^2 y\,dy$

(C) $\pi\int_0^1 e^{4x}\,dx$



$y = e^{2x}$

$\ln y = 2x$

$\frac{\ln y}{2} = x$

$\pi\int_0^1 1\,dy + \pi\int_1^{e^2} 1-\left(\frac{\ln y}{2}\right)^2 dy$

$\pi + \pi\int_1^{e^2} 1 - \frac{(\ln y)^2}{4}\,dy$

13.177

$2\pi\int_0^1 xe^{2x}\,dx$

10. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) −2

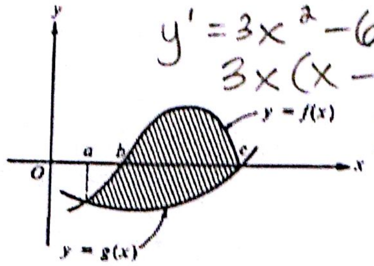
$y' = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$

x	y
−2	−8
0	12
2	8
4	28

11.



The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)\,dx$ (D) $\int_a^c (f(x) - g(x))\,dx$

(B) $\int_b^c f(x)\,dx - \int_a^c g(x)\,dx$ (E) $\int_a^b (g(x) - f(x))\,dx + \int_b^c (f(x) - g(x))\,dx$

(C) $\int_a^c (g(x) - f(x))\,dx$

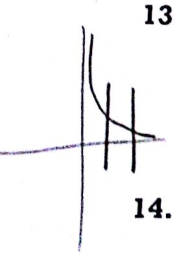
$\int_3^4 \left(\frac{1}{x-1}\right) dx$

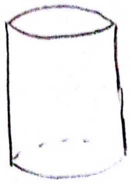
$\int_0^3 (3x - x^2)\,dx$

12. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) −6 (B) −2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

13. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?





$V = \pi r^2 h = 16\pi$

$h = \frac{16}{r^2}$

$r^2 = \frac{16}{h}$

$\sqrt[3]{16}$ $\sqrt[3]{32}$

(A) $2\sqrt[3]{2}$ $(16)^{1/3}$ (B) $2\sqrt{2}$ $\sqrt{8}$ (C) $2\sqrt[3]{4}$ $(32)^{1/3}$ (D) 4 (E) 8

$A = 2\pi rh + 2\pi r^2$ $r = 4h^{-1/2}$

$A = 2\pi(4h^{-1/2})(h) + 2\pi(4h^{-1/2})^2$

$A =$ → on back

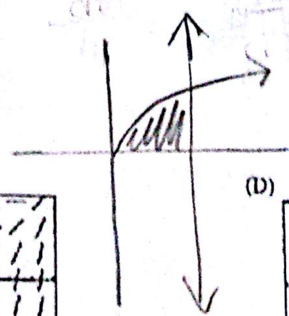
14. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

$\pi\int_0^3 (\sqrt{x})^2\,dx$

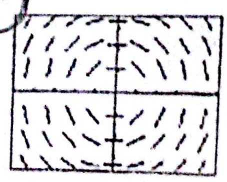
15. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

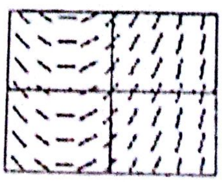


16. Match the slope field with the differential equation: $\frac{dy}{dx} = -\frac{x}{y}$

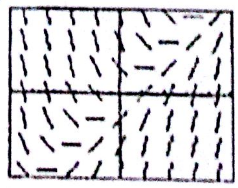
(A)



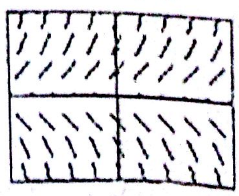
(B)



(C)



(D)



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1 : description

$$A = 8\pi h^{1/2} + 32\pi h^{-1}$$

$$\frac{dA}{dh} = 4\pi h^{-1/2} - 32\pi h^{-2} = 0$$

$$4\pi\left(\frac{1}{\sqrt{h}} - \frac{8}{h^2}\right) = 0$$

$$6: \begin{cases} 1: \text{separates variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \\ \quad 0/1 \text{ if } y \text{ is not exponential} \end{cases}$$

$$\frac{1}{\sqrt{h}} = \frac{8}{h^2}$$

$$h = 4$$

$$\frac{1}{2} = \frac{8}{16}$$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables