1. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

2. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

3. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

4. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the <u>y-axis</u>. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

5. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

6. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

7. $\int_1^4 |x-3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

8. The $\lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

9. A region in the first quadrant is enclosed by the graphs of $y=e^{2x}$, $x=1$, and the coordinate axes. If the region is rotated about the <u>y-axis</u>, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi\int_0^1 xe^{2x}\,dx$ (D) $\pi\int_0^e y\ln y\,dy$

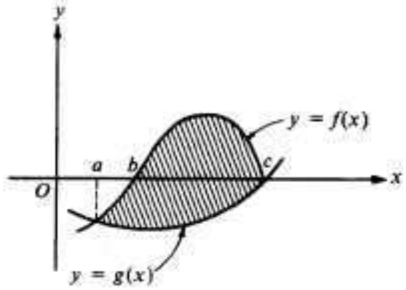
(B) $2\pi\int_0^1 e^{2x}\,dx$ (E) $\frac{\pi}{4}\int_0^e \ln^2 y\,dy$

(C) $\pi\int_0^1 e^{4x}\,dx$

10. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) −2

11.



The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)|-|g(x)|)\,dx$ (D) $\int_a^c (f(x)-g(x))\,dx$

(B) $\int_b^c f(x)\,dx-\int_a^c g(x)\,dx$ (E) $\int_a^b (g(x)-f(x))\,dx+\int_b^c (f(x)-g(x))\,dx$

(C) $\int_a^c (g(x)-f(x))\,dx$

12. What is the average value of y for the part of the curve $y=3x-x^2$ which is in the <u>first quadrant</u>?

(A) −6 (B) −2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

13. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

14. The area of the region enclosed by the curve $y=\frac{1}{x-1}$, the x-axis, and the lines $x=3$ and $x=4$ is

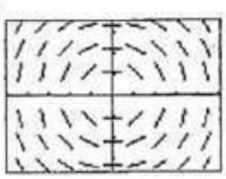
(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

15. The region enclosed by the x-axis, the line $x=3$, and the curve $y=\sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

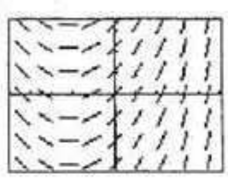
(A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

16. Match the slope field with the differential equation: $\frac{dy}{dx}=-\frac{x}{y}$

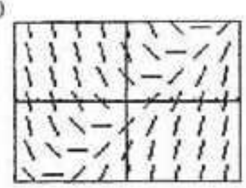
(A)



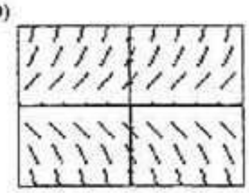
(B)



(C)



(D)

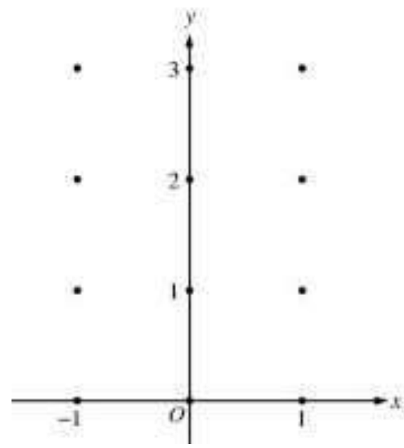


AP CALCULUS AB **Problem Set Unit 6** **Name: ____________________________**

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

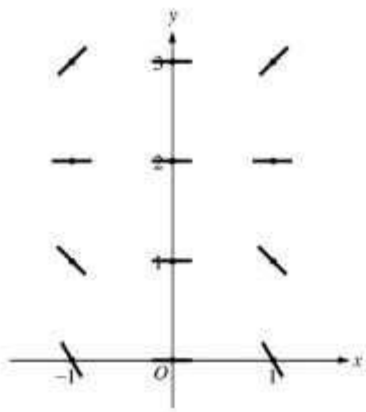
ANSWERS: '69#13,25, '73 #17, '85 #45, '88 #21, 24, 28, 29, 30, 33, 34, 36, 45, '93 # 6, 30, random sheet

2004 Form B

1. C	2. A	3. C	4. A	5. C	6. C	7. C	8. B
9. A	10. A	11. D	12. C	13. D	14. D	15. C	16. A

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a)



2 : { 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
1 : { positive slope at each point (x, y) where $x \neq 0$ and $y > 2$; negative slope at each point (x, y) where $x \neq 0$ and $y < 2$ }

(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

1 : description

(c) $\frac{1}{y-2}dy = x^4 dx$

$\ln|y-2| = \frac{1}{5}x^5 + C$

$|y-2| = e^C e^{\frac{1}{5}x^5}$

$y - 2 = Ke^{\frac{1}{5}x^5}, \; K = \pm e^C$

$-2 = Ke^0 = K$

$y = 2 - 2e^{\frac{1}{5}x^5}$

6 : { 1 : separates variables
2 : antiderivatives
1 : constant of integration
1 : uses initial condition
1 : solves for y
0/1 if y is not exponential }

Note: max 3/6 [1-2-0-0-0] if no constant of integration
Note: 0/6 if no separation of variables