Free Response for Practice

1991 BC1

A particle moves on the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of t for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

1991 BC1
Solution

(a) $x(t) = 4t^3 - 18t^2 + 15t + C$

$0 = x(1) = 4 - 18 + 15 + C$

Therefore $C = -1$

$x(t) = 4t^3 - 18t^2 + 15t - 1$

(b) $0 = v(t) = 12t^2 - 36t + 15$

$3(2t-1)(2t-5) = 0$

$t = \frac{1}{2}, \frac{5}{2}$

(c) $\frac{dv}{dt} = 24t - 36$

$\frac{dv}{dt} = 0$ when $t = \frac{3}{2}$

$v(0) = 15$

$v\left(\frac{3}{2}\right) = -12$

$v(2) = -9$

Maximum velocity is 15

(d) Total distance $= \int_0^{1/2} v(t)\,dt - \int_{1/2}^{2} v(t)\,dt$

$$= \left(x\left(\frac{1}{2}\right) - x(0)\right) - \left(x(2) - x\left(\frac{1}{2}\right)\right)$$

$$= \frac{5}{2} - (-1) - \left(-11 - \frac{5}{2}\right) = 17$$

1992 AB2

A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

(a) Find the minimum acceleration of the particle.

(b) Find the total distance traveled by the particle.

(c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

1992 AB2
Solution

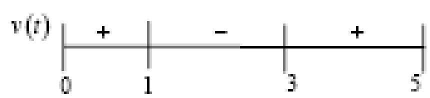
(a) $v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

a is increasing, so a is minimum at $t = 0$

$a(0) = -12$ is minimum value of a.

(b) Method 1:



$$d = \int_0^1 (3t^2 - 12t + 9)\,dt - \int_1^3 (3t^2 - 12t + 9)\,dt + \int_3^5 (3t^2 - 12t + 9)\,dt$$

$$= \left[t^3 - 6t^2 + 9t\right]_0^1 - \left[t^3 - 6t^2 + 9t\right]_1^3 + \left[t^3 - 6t^2 + 9t\right]_3^5$$

$$= 4 - (-4) + 20 = 28$$

or

Method 2: $x(t) = t^3 - 6t^2 + 9t - 2$

$\left[\text{or } x(t) = t^3 - 6t^2 + 9t + C\right]$

$x(0) = -2$

$x(1) = 2$

$x(3) = -2$

$x(5) = 18$

Total distance $= 4 + 4 + 20 = 28$

(c) Method 1:

$$\frac{\int_0^5 (3t^2 - 12t + 9)\,dt}{5 - 0}$$

$$= \frac{1}{5}\left[t^3 - 6t^2 + 9t\right]_0^5 = \frac{1}{5}(20) = 4$$

or

Method 2: $\dfrac{x(5) - x(0)}{5 - 0} = \dfrac{18 - (-2)}{5} = 4$

1997 AB1

A particle moves along the x-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \ge 0$.

(b) For what values of t, $0 \le t \le 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0,3]$?

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

1997 AB1
Solution

(a) $x(t) = \int v(t)\,dt = \int (3t^2 - 2t - 1)\,dt$

$= t^3 - t^2 - t + C$

$x(2) = 8 - 4 - 2 + C = 5; \quad C = 3$

$x(t) = t^3 - t^2 - t + 3$

(b) avg. vel. $= \dfrac{x(3) - x(0)}{3 - 0}$

$= \dfrac{18 - 3}{3} = 5$

$3t^2 - 2t - 1 = 5$

$t = \dfrac{1 + \sqrt{19}}{3}$ or 1.786

(c) distance $= \int_0^3 |v(t)|\,dt$

$= \int_0^3 |3t^2 - 2t - 1|\,dt = 17$

or

$v(t) = 3t^2 - 2t - 1 = 0$

$t = -\frac{1}{3},\ t = 1$

$x(0) = 3$

$x(1) = 1 - 1 - 1 + 3 = 2$

$x(3) = 27 - 9 - 3 + 3 = 18$

distance $= (3 - 2) + (18 - 2) = 17$

The table shows the depth of water, W, in a river, as measured at 4-hour intervals during a day-long flood. Assume that W is a differential function of time t.

t (hr)	0	4	8	12	16	20	24
W(t) (ft)	32	36	38	37	35	33	32

a) Find the approximate value of W'(16). Indicate units of measure
b) Estimate the average depth of the water, in feet, over the time interval [0,24] hours by using a trapezoidal approximation with subintervals of length Δt = 4 days.
c) Scientists studying the flooding believe they can model the depth of the water with the function $F(t) = 35 - 3cos\left(\frac{t+3}{4}\right)$, where F(t) represents the depth of the water, in feet, after t hours. Find F'(16) and explain the meaning of your answer, with appropriate units, in terms of the river depth.
d) Use the function F to find the average depth of the water, in feet, over the time interval [0, 24] hours

Solution

a) W'(16)=$\frac{W(20)-W(16)}{20-16} = \frac{33-35}{4} = \frac{1}{2}ft/hr$
b) The average value of a function is the integral across the given interval divided by the interval width. Here $Avg(W) = \frac{\int_0^{24} W(t)dt}{24-0}$. Estimate the value of the integral using trapezoid rule T with values from the table and Δt = 4:

$$T = \frac{4}{2}(32 + 2(36) + 2(38) + 2(37) + 2(35) + 2(33) + 32) = 844$$

Therefore the Avg(W)=844/24 = 35.167 ft
c) Using your calculator to evaluate, F'(16) = 0.749. After 16 days, the river depth is dropping at the rate of 0.749 ft/hr.
d) Avg(F) = 35.116 ft (again use your calculator)

During a recent snowfall, several students monitored the accumulation of snow on the flat roof of their school. The table records the data they collected for the 12-hour period of the snowfall.

# of hours	Rate of snowfall (in/hour)
0	0
2	1.5
3	2.1
4.5	2.4
6.5	2.8
8	2.2
10.5	1.8
12	1.6

a) Use a right-hand sum to approximate the total depth of snow in the 12-hour period.
b) Using the right-hand sum approximation, estimate the average rate of snowfall in the 12-hour period.

Solution

a) 2(1.5) + 1(2.1) +1.5(2.4) + 2(2.8) + 1.5(2.2) +2.5(1.8) + 1.5(1.6) = 24.5 in
b) 24.5/12 = 2.042 in/hour