5.2

(1) $\int \frac{1}{3x+2}\,dx$

$u = 3x+2$
$du = 3\,dx$
$\frac{1}{3}du = dx$

$\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|3x+2| + C$

(2) $\int \frac{x(x+2)}{x^3+3x^2-4}\,dx$

$\int \frac{x^2+2x}{x^3+3x^2-4}\,dx$

$u = x^3+3x^2-4$
$du = (3x^2+6x)\,dx$
$du = 3(x^2+2x)\,dx$
$\frac{1}{3}du = (x^2+2x)\,dx$

$\frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln|x^3+3x^2-4| + C$

(3) $\int \frac{x^3-6x-20}{x+5}\,dx$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -6 & -20 \\ & \downarrow & -5 & 25 & -95 \\ \hline & 1 & -5 & 19 & -115 \end{array}$$

$x^2-5x+19-\frac{115}{x+5}$

$\int \left(x^2-5x+19-\frac{115}{x+5}\right)dx$

$= \frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x+5| + C$

(4) $\int \frac{1}{x\cdot\ln(x^3)}\,dx$

$u = \ln(x^3)$
$du = \left(\frac{1}{x^3}\cdot 3x^2\right)dx$
$du = \frac{3}{x}\,dx$
$\frac{1}{3}du = \frac{1}{x}\,dx$

$\frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln|\ln(x^3)| + C$

⑤ $\int_0^1 \frac{x-1}{x+1}\,dx$ $\quad u=x+1 \quad du=dx \quad u(0)=1 \quad u(1)=2$

$x=u-1$

$$\int_1^2 \frac{u-1-1}{u}\,du = \int_1^2 \frac{u-2}{u}\,du = \int_1^2 \left(1-\frac{2}{u}\right)du$$

$$= u - 2\ln|u|\Big|_1^2 = (2-2\ln 2)-(1-\cancel{2\ln 1}^{\,0})$$

$$2-2\ln 2 - 1$$

$$1-2\ln 2$$

⑥ $\int e^{3x+1}\,dx$ $\quad u=3x+1 \quad du=3dx \quad \frac{1}{3}du=dx$

$$\frac{1}{3}\int e^u\,du$$

$$=\frac{1}{3}e^{3x+1}+C$$

⑦ $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$ $\quad u=x^{-1} \quad du=-x^{-2}dx \quad du=-\frac{1}{x^2}dx \quad -du=\frac{1}{x^2}dx$

$$-\int e^u\,du$$

$$=-e^{\frac{1}{x}}+C$$

⑧ $\int 5xe^{-x^2}\,dx$ $\quad u=-x^2 \quad du=-2x\,dx \quad -\frac{1}{2}du=x\,dx$

$$5\cdot -\frac{1}{2}\int e^u\,du$$

$$=-\frac{5}{2}e^{-x^2}+C$$

⑨ $\int_0^1 \frac{e^x}{1+e^x}\,dx$ $\quad u=1+e^x \quad du=e^x dx \quad u(0)=2 \quad u(1)=1+e$

$$\int_2^{1+e} \frac{1}{u}\,du = \ln|u|\Big|_2^{1+e}$$

$$\ln|1+e| - \ln 2 = \ln\left(\frac{1+e}{2}\right)$$

⑩ $\int \frac{2e^x-2e^{-x}}{(e^x+e^{-x})^2}\,dx$ $\quad u=e^x+e^{-x} \quad du=(e^x-e^{-x})dx$

$$2\cdot\int \frac{1}{u^2}\,du = 2\cdot -u^{-1}+C = \frac{-2}{e^x+e^{-x}}+C$$

(11) $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$

$F'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

(12) $\int_1^4 \frac{x+4}{x}\,dx = \int_1^4 \left(1 + \frac{4}{x}\right) dx = x + 4\ln|x| \Big|_1^4$

$(4 + 4\ln 4) - (1 + \cancel{4\ln 1})$

$4 + 4\ln 4 - 1$

$\left.\begin{array}{l} = 3 + 4\ln 4 \\ = 3 + \ln 4^4 \\ = 3 + 4\ln 2^2 \\ = 3 + 8\ln 2 \end{array}\right\}$ all $=$

(13) $\frac{1}{2-0} \cdot \int_0^2 \sec\left(\frac{\pi}{6}x\right) dx$

$u = \frac{\pi}{6}x$ $\quad u(0) = 0$

$du = \frac{\pi}{6}dx$ $\quad u(2) = \pi/3$

$\frac{6}{\pi}du = dx$

$\frac{1}{2} \cdot \frac{6}{\pi} \int_0^{\pi/3} \sec u\,du$

$\frac{3}{\pi} \cdot \ln|\sec u + \tan u| \Big|_0^{\pi/3}$

$\frac{3}{\pi} \cdot \left[\ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln|\sec 0 + \tan 0|\right]$

$\frac{3}{\pi}\left[\ln(2 + \sqrt{3}) - \cancel{\ln(1 + 0)}^{\,0}\right]$

$\frac{3}{\pi} \ln(2 + \sqrt{3})$

(14) $f''(x) = \sin x + e^{2x}$ $\quad f'(0) = \frac{1}{2}$ $\quad f(0) = \frac{1}{4}$

$$f'(x) = \int (\sin x + e^{2x})\,dx$$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$$f'(x) = -\cos x + \frac{1}{2}e^{2x} + C$$

$$\frac{1}{2} = -\cos 0 + \frac{1}{2}e^{0} + C$$

$$\frac{1}{2} = -1 + \frac{1}{2} + C$$

$$C = 1$$

$$f'(x) = -\cos x + \frac{1}{2}e^{2x} + 1$$

$$f(x) = \int \left(-\cos x + \frac{1}{2}e^{2x} + 1\right) dx$$

$$f(x) = -\sin x + \frac{1}{4}e^{2x} + x + C$$

$$\frac{1}{4} = \cancel{-\sin 0} + \frac{1}{4}e^{0} \cancel{+ 0} + C$$

$$\frac{1}{4} = \frac{1}{4} + C$$

$$C = 0$$

$$f(x) = -\sin x + \frac{1}{4}e^{2x} + x$$

Scanned by CamScanner