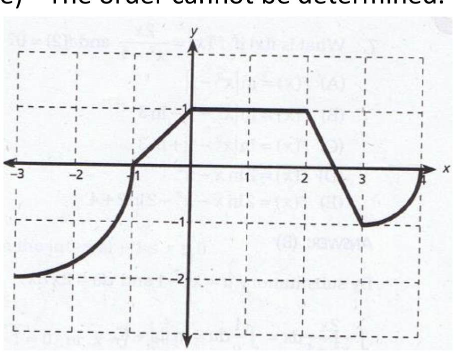
Problem Set Four – Integration Name: ______________________
AP Calculus AB Date: ______________________

Multiple Choice: No Calculator unless otherwise indicated.

1) $\int sec^2x - 2dx$
   a) tan x + C
   b) tan x – 2x +C
   c) $\frac{tan^3x}{3} - x + C$
   d) $2tanxsec^2x - x + C$
   e) None of these

2) If F(x)=$\int_0^{x^2} \sqrt{t+3}dt$, what is F'(x)
   a) $\sqrt{x^2+3}$
   b) $\frac{1}{2\sqrt{x^2+3}}$
   c) $2x\sqrt{x^2+3}$
   d) $\frac{2(x^2+3)}{3}$
   e) None of these

3) Let $A = \int_0^1 cosxdx$. We estimate A using the LRAM, RRAM and Trapezoidal approximations with n = 100 subintervals. Which is true?
   a) L < A < T < R
   b) L < T < A < R
   c) R < A < T < L
   d) R < T < A < L
   e) The order cannot be determined.



4) The graph of f(x) consists of line segments and quarter circles as sown in the graph above. What is the value of $\int_{-3}^{4} f(x)dx$?
   a) $\frac{10-5\pi}{4}$
   b) $\frac{10+5\pi}{4}$
   c) $\frac{12+5\pi}{4}$
   d) $\frac{12-5\pi}{4}$
   e) None of these

5) Let R be the region between the function $f(x) = x^3 + 6x^2 + 10x + 4$, the x-axis and the lines x = 0 and x = 4. Using the Trapezoidal Rule, compute the area when there are 4 equal subdivisions. (Calculator)
   a) 196
   b) 288
   c) 296
   d) 396
   e) None of these

6) What is f(x) if f'(x) = $\frac{2x}{x^2-1}$ and f(2) = 0 (Calculator)
   a) $f(x) = \ln|x^2 - 1|$
   b) $f(x) = \ln|x^2 - 1| - ln3$
   c) $f(x) = \ln|x^2 - 1| + ln3$
   d) $f(x) = 2lnx - x^2$
   e) $f(x) = 2lnx - x^2 - 2ln2 + 4$

7) What value of c on the closed interval [1, 3] satisfies the Mean Value Theorem for Integrals for $f(x) = 2lnx$? (calculator)
   a) 2.592
   b) 2.000
   c) 1.912
   d) 1.296
   e) None of theses

8) Find the value of x at which the function $y = x^2$ reaches its average value on the interval [0, 10] (calculator)
   a) 4.642
   b) 5
   c) 5.313
   d) 5.774
   e) 7.071

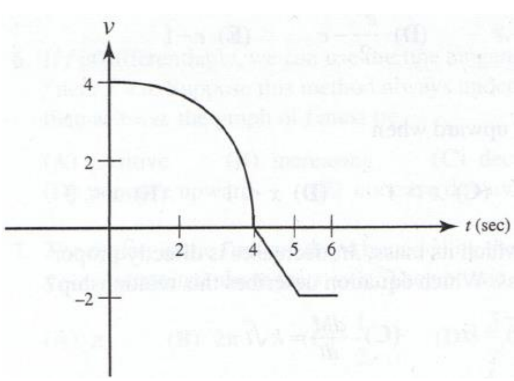
9) If $\int_1^3 f(x)dx = k$ and $\int_1^7 f(x)dx = -4$, what is the value of $\int_7^3 x + f(x)dx$?
   a) k + 4
   b) k – 4
   c) 16 – k
   d) -16 + k
   e) -16 - k

Problem Set Four – Integration Name: ______________________
AP Calculus AB Date: ______________________

10) If a particle is moving in a straight line with a velocity of $v(t) = 2t - 3$ ft/sec and its position at t = 2 sec is -10ft, find its position at t = 5 sec.
   a) -22 ft
   b) 2 ft
   c) 10 ft
   d) 12 ft
   e) 22 ft

11) Let $F(x) = \int_0^x \frac{10}{1+e^t}\,dt$. Which of the following statements is/are true? (calculator)
   I. F'(0)=5
   II. F(2)<F(6)
   III. F is concave upward
   a) I only
   b) II only
   c) III only
   d) I and II only
   e) I and III only

Questions 12 & 13. The graph below consists of a quarter circle and two line segments and represents the velocity of an object during a 6-second interval.



12) The object's average speed in (units/sec) during the 6-second interval is
   a) $\frac{4\pi+3}{6}$
   b) $\frac{4\pi-3}{6}$
   c) -1
   d) -1/3
   e) 1

13) The object's acceleration (units/sec^2) at t = 4.5 is
   a) 0
   b) -1
   c) -2
   d) -1/4
   e) 4π – 1/4

14) Using midpoint approximation with three sub-intervals, what is the approximate area of the function, $y = 6x - x^2$, on the interval [0, 6].
   a) 9
   b) 19
   c) 36
   d) 38
   e) 54

15) Using a Trapezoid approximation, what is the approximate area of the above function using 6 subintervals?
   a) 17.5
   b) 30
   c) 35
   d) 36
   e) 6

Problem Set Four – Integration Name: _________________________
AP Calculus AB Date: _________________________

Answer Sheet

1) B	2) C	3) D	4) A	5) C
6) B	7) C	8) D	9) D	10) B
11) D	12) A	13) c	14) D	15) C

Free Response question #1

2 points 1 point for FTC antiderivative 1 point for answer The integral can be computed using the FTC and some basic properties $\int_{-1}^{0}(2f'(x)+3f''(x))dx = 2f(0)-2f(-1)+3f'(0)-3f'(-1)=13$
3 points 1 point for tangent line 1 point for approximation 1 point for "less" with reason y – 3 = -3(x+2). The tangent line is a linear approximation for f near (-2, 3) so $f(-2.1) \approx 3(-2.1) - 3 = 3.3$ Since f is concave up for -2.1 < x < 2, the actual value of f(2.1) is greater than 3.3
2 points 1 point for MVT reference 1 point for value of r $g'(c) = \frac{g(b)-g(a)}{b-a}$ Since f' is differentiable on (0, 1), there exists a point c, with 0 < c < 1, such that $f''(c) = \frac{f'(1)-f'(0)}{1-0} = \frac{5-3}{1} = 2$ So r = 2 works.
2 points 1 point for "no" with reference to MVT 1 point for correct reasoning Using the MVT on the interval (-1, 0) we can guarantee the existence of b with -1 < b < 0, such that $f''(b) = \frac{f'(0)-f'(-1)}{0-(-1)} = \frac{3-0}{1} = 3$ From part c we know that for some c with 0 < c < 1, f''(b)=2. Since b < c and f"(b) > f"(c), the function cannot be increasing for all x on the interval (-1, 1).

Problem Set Four – Integration   Name: ____________________________
AP Calculus AB   Date: ____________________________

FR Question #2

2 points 1 point for setting up the sum 1 point for the answer with units 5(5) + 5(20) + 10(30) + 10(15) + 5(0) = 575 GAL
3 points 1 point for explanation 1 use of Reimann sum 1 answer $\frac{1}{20}\int_{10}^{30} R(t)dt$ is the average rate of change of the leak over the 20-minute period, $10 \le t \le 30$. $\frac{1}{20}\int_{10}^{30} R(t)dt = \frac{1}{20}\left[10(30)+10(15)\right] = 22.5 gal/\min$
$R'(25) = \frac{R(30)-R(20)}{30-20} = \frac{15-30}{10} = -1.5 gal/min$ There are multiple answers (2 points)
2 Points 1 Point for setting Q'(t) = 0 1 point for answer with reasoning T = 18.805 min $Q''(t) = -16.78 \sin(0.15t - 1.25)(.15)^2$ And Q"(18.805) < 0 thus the leak is at a maximum rate at t = 18.805 min.