Problem Set Four – Integration
AP Calculus AB

Name: ____________________________
Date: ___________________________

Multiple Choice: No Calculator unless otherwise indicated.

1) \( \int \sec^2 x - 2 \, dx \)
   a) \( \tan x + C \)
   b) \( \tan x - 2x + C \)
   c) \( \frac{\tan^3 x}{3} - x + C \)
   d) \( 2\tan x \sec^2 x - x + C \)
   e) None of these

2) If \( F(x) = \int_0^\infty \sqrt{t^2 + 3} \, dt \), what is \( F'(x) \)
   a) \( \sqrt{x^2 + 3} \)
   b) \( \frac{1}{2\sqrt{x^2 + 3}} \)
   c) \( 2x\sqrt{x^2 + 3} \)
   d) \( \frac{2(x^2 + 3)}{3} \)
   e) None of these

3) Let \( A = \int_0^1 \cos x \, dx \). We estimate \( A \) using the LRAM, RRAM and Trapezoidal approximations with \( n = 100 \) subintervals. Which is true?
   a) \( L < A < T < R \)
   b) \( L < T < A < R \)
   c) \( R < A < T < L \)
   d) \( R < T < A < L \)
   e) The order cannot be determined.

4) The graph of \( f(x) \) consists of line segments and quarter circles as sown in the graph above. What is the value of \( \int_{-3}^{4} f(x) \, dx \)?
   a) \( \frac{10 - 5\pi}{4} \)
   b) \( \frac{10 + 5\pi}{4} \)
   c) \( \frac{12 + 5\pi}{4} \)
   d) \( \frac{12 - 5\pi}{4} \)
   e) None of these

5) Let \( R \) be the region between the function \( f(x) = x^3 + 6x^2 + 10x + 4 \), the \( x \)-axis and the lines \( x = 0 \) and \( x = 4 \). Using the Trapezoidal Rule, compute the area when there are 4 equal subdivisions.
   (Calculator)
   a) 196
   b) 288
   c) 296
   d) 396
   e) None of these

6) What is \( f(x) \) if \( f'(x) = \frac{2x}{x^2 - 1} \) and \( f(2) = 0 \) (Calculator)
   a) \( f(x) = \ln |x^2 - 1| \)
   b) \( f(x) = \ln |x^2 - 1| - \ln 3 \)
   c) \( f(x) = \ln |x^2 - 1| + \ln 3 \)
   d) \( f(x) = 2\ln x - x^2 \)
   e) \( f(x) = 2\ln x - x^2 - 2\ln 2 + 4 \)

7) What value of \( c \) on the closed interval \([1, 3]\) satisfies the Mean Value Theorem for Integrals for \( f(x) = 2\ln x \)? (calculator)
   a) 2.592
   b) 2.000
   c) 1.912
   d) 1.296
   e) None of these

8) Find the value of \( x \) at which the function \( y = x^2 \) reaches its average value on the interval \([0, 10]\) (calculator)
   a) 4.642
   b) 5
   c) 5.313
   d) 5.774
   e) 7.071

9) If \( \int_1^3 f(x) \, dx = k \) and \( \int_1^7 f(x) \, dx = -4 \), what is the value of \( \int_7^x x + f(x) \, dx \)?
   a) \( k + 4 \)
   b) \( k - 4 \)
   c) \( 16 - k \)
   d) \(-16 + k \)
   e) \(-16 - k \)
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10) If a particle is moving in a straight line with a
velocity of \( v(t) = 2t - 3 \) ft/sec and its position at \( t = 2 \) sec is -10 ft, find its position at \( t = 5 \) sec.
   a) -22 ft
   b) 2 ft
   c) 10 ft
   d) 12 ft
   e) 22 ft

11) Let \( F(x) = \int_0^x \frac{10}{1+e^t} \, dt \). Which of the following statements is/are true? (calculator)
   I. \( F'(0) = 5 \)
   II. \( F(2) < F(6) \)
   III. \( F \) is concave upward
   a) I only
   b) II only
   c) III only
   d) I and II only
   e) I and III only

Questions 12 & 13. The graph below consists of a quarter circle and two line segments and represents the velocity of an object during a 6-second interval.

12) The object’s average speed in (units/sec) during the 6-second interval is
   a) \( \frac{4\pi + 3}{6} \)
   b) \( \frac{4\pi - 3}{6} \)
   c) -1
   d) -1/3
   e) 1

13) The object’s acceleration (units/sec\(^2\)) at \( t = 4.5 \) is
   a) 0
   b) -1
   c) -2
   d) -1/4
   e) \( 4\pi - 1/4 \)

14) Using midpoint approximation with three sub-intervals, what is the approximate area of the function, \( y = 6x - x^2 \), on the interval \([0, 6]\).
   a) 9
   b) 19
   c) 36
   d) 38
   e) 54

15) Using a Trapezoid approximation, what is the approximate area of the above function using 6 subintervals?
   a) 17.5
   b) 30
   c) 35
   d) 36
   e) 6
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Answer Sheet

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<tbody>
<tr>
<td>1) B</td>
<td>2) C</td>
<td>3) D</td>
<td>4) A</td>
<td>5) C</td>
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<tr>
<td>6) B</td>
<td>7) C</td>
<td>8) D</td>
<td>9) D</td>
<td>10) B</td>
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Free Response question #1

2 points
1 point for FTC antiderivative
1 point for answer

The integral can be computed using the FTC and some basic properties
\[ \int_{-1}^{0} (2f'(x) + 3f''(x)) \, dx = 2f(0) - 2f(-1) + 3f'(0) - 3f'(-1) = 13 \]

3 points
1 point for tangent line
1 point for approximation
1 point for “less” with reason

The tangent line is a linear approximation for \( f \) near \((-2, 3)\) so
\[ f(-2.1) \approx 3(-2.1) - 3 = 3.3 \]

Since \( f \) is concave up for \(-2.1 < x < 2\), the actual value of \( f(2.1) \) is greater than 3.3

2 points
1 point for MVT reference
1 point for value of \( r \)

\[ g'(c) = \frac{g(b) - g(a)}{b - a} \]

Since \( f' \) is differentiable on \((0, 1)\), there exists a point \( c \), with \( 0 < c < 1 \), such that
\[ f''(c) = \frac{f'(1) - f'(0)}{1 - 0} = \frac{5 - 3}{1} = 2 \]

So \( r = 2 \) works.

2 points
1 point for “no” with reference to MVT
1 point for correct reasoning

Using the MVT on the interval \((-1, 0)\) we can guarantee the existence of \( b \) with \(-1 < b < 0\), such that
\[ f''(b) = \frac{f'(0) - f'(-1)}{0 - (-1)} = \frac{3 - 0}{1} = 3 \]

From part c we know that for some \( c \) with \( 0 < c < 1 \), \( f''(b) = 2 \). Since \( b < c \) and \( f''(b) > f''(c) \), the function cannot be increasing for all \( x \) on the interval \((-1, 1)\).
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FR Question #2

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<tr>
<th>2 points</th>
<th>1 point for setting up the sum</th>
<th>1 point for the answer with units</th>
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<tbody>
<tr>
<td></td>
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<td>$5(5) + 5(20) + 10(30) + 10(15) + 5(0) = 575$ GAL</td>
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<tr>
<th>3 points</th>
<th>1 point for explanation</th>
<th>1 use of Reimann sum</th>
<th>1 answer</th>
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<tr>
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<td>$\frac{1}{20} \int_{10}^{30} R(t) , dt$ is the average rate of change of the leak over the 20-minute period, $10 \leq t \leq 30.$</td>
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<td>$\frac{1}{20} \int_{10}^{30} R(t) , dt = \frac{1}{20} [10(30) + 10(15)] = 22.5$ gal/ min</td>
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$R'(25) = \frac{R(30) - R(20)}{30 - 20} = \frac{15 - 30}{10} = -1.5$ gal/min  
There are multiple answers (2 points)

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<tr>
<th>2 Points</th>
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<th>1 point for answer with reasoning</th>
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<tr>
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<td>$T = 18.805$ min</td>
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<td>$Q''(t) = -1.678 \sin (0.15t - 1.25)(.15)^2$</td>
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<td>And $Q''(18.805) &lt; 0$ thus the leak is at a maximum rate at $t = 18.805$ min.</td>
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