Prob Set 4

1) B

$$\int (\sec^2 x - 2)\,dx = \tan x - 2x + C$$

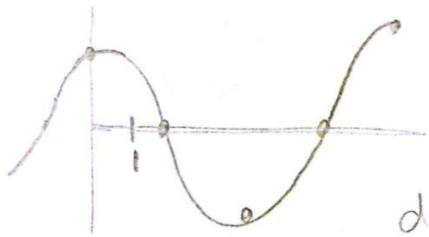
2) C

$$F(x) = \int_0^{x^2} \sqrt{t+3}\,dt$$

$$F'(x) = \sqrt{x^2+3} \cdot 2x$$

3) D

$$A = \int_0^1 \cos x\,dx$$



decreasing

cc↓

LRAM above

RRAM below

TRAM below

4) A

$$\int_{-3}^{4} f(x)\,dx =$$

$$-\frac{1}{4}\pi(2)^2 = -\pi$$

$$\frac{1}{2}(1)(1) = \frac{1}{2}$$

$$2(1) = 2$$

$$-\frac{1}{4}\pi(1)^2 = \frac{-\pi}{4}$$

$$-\frac{4\pi}{4} - \frac{\pi}{4} + \frac{8}{4} + \frac{2}{4}$$

$$\frac{10 - 5\pi}{4}$$

5) $\frac{1}{2}(1)(4+2(21)+2(56)+2(115)+204)$

C $f(0)=4$ $=296$

$f(1)=21$

$f(2)=56$

$f(3)=115$

$f(4)=204$

6) $\int \frac{2x}{x^2-1}\,dx$ $u=x^2-1$

$du=2x\,dx$

B

$\int \frac{1}{u}\,du = \ln|u|+C$

$0=\ln|2^2-1|+C$

$0=\ln 3+C$ $C=-\ln 3$

$\ln|x^2-1|-\ln 3$

7) $f(c)=\frac{1}{3-1}\int_1^3 2\ln x\,dx$

C

$\frac{1}{2}\cdot 2\int_1^3 \ln x\,dx$

$=\int_1^3 \ln x\,dx$

$f(c)=1.296=2\ln x$

$.648=\ln x$

$x=1.912$

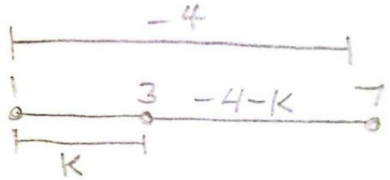
$f(c) = \frac{1}{10}\int_0^{10} x^2\,dx = \frac{100}{3} = f(c)$

D

$\frac{100}{3} = x^2$

9)

D



$\int_7^3 (x+f(x))\,dx$

$-\int_3^7 (x+f(x))\,dx$

$-\left(\frac{x^2}{2}\Big|_3^7 + (-4-K)\right)$

$-\left[\left(\frac{49}{2}-\frac{9}{2}\right)+(-4-K)\right]$

$20 + {}^-4 - K$

$-(16-K)$

$-16+K$

10) $v(t) = 2t-3$

B

$t=2,\ s(t) = -10$

$s(5) =$

$\int (2t-3)\,dt = t^2 - 3t + C$

$-10 = 2^2 - 3(2) + C$

$-10 = 4 - 6 + C$

$-10 = -2 + C$

$-8 = C$

$s(t) = t^2 - 3t - 8$

$s(5) = 25 - 15 - 8$

$= 2$

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11) $F(x) = \int_0^x \frac{10}{1+e^t}\,dt$

D

$F'(x) = \frac{10}{1+e^x} = \frac{10}{1+1} = 5$

I true

$\int_0^2 \frac{10}{1+e^t}\,dt \approx 5.662$ $\int_0^6 \frac{10}{1+e^t}\,dt \approx 6.907$

II true

$F'(x) = f(x)$

$F''(x) = f'(x)$ III false

12) $\frac{1}{6-0} \cdot \int_0^6 |v(t)|\,dt = \frac{1}{6} \cdot \left(\frac{1}{4}\pi(4)^2 + \frac{1}{2}(2)(2+1)\right)$

A

$\frac{1}{6}(4\pi + 3)$

$\frac{4\pi+3}{6}$

13) -2

C

14) 0 1 2 3 4 5 6

5 9 5

D

$2(5) + 2(9) + 2(5) = 38$

15) $\frac{1}{2}(1)\left(0 + 2(5) + 2(8) + 2(9) + 2(8) + 2(5) + 0\right) = 35$

C

PROB SET FRQ 1

a) $\int_{-1}^{0}(2f'(x)+3f''(x))dx$

$2\int_{-1}^{0} f'(x)dx + 3\int_{-1}^{0} f''(x)dx$

$2\left(f(x)\Big|_{-1}^{0} + 3f'(x)\Big|_{-1}^{0}\right)$

$2(f(0)-f(-1))+3(f'(0)-f'(-1))$

$2(3-1)+3(3-0)$

$2(2)+3(3)=4+9=13$

b) $(-2,3)$

$f'(-2)=-3$

$y-3=-3(x+2)$

$f(-2.1) \approx -3(-2.1+2)+3=3.3$

less than the actual because f is cc↑

c) $g'(c)=\dfrac{g(b)-g(a)}{b-a}$

There exists a point c, $0<c<1$, such that

(MVT) $f''(c)=\dfrac{f'(1)-f'(0)}{1-0}=\dfrac{5-3}{1}=2$

$r=2$

d) By MVT, there exists a value, b, on $-1<b<0$, such that

$$f''(b)=\frac{f'(0)-f'(-1)}{0-(-1)}=3$$

From part c, we know $f''(b)=2$. Since $b<c$ and $f''(b)>f''(c)$, the function can't be increasing for all x on $(-1,1)$.

FRQ #2

a) $5(5)+5(20)+10(30)+10(15)+5(0)=$ 575 gallons

b) $\frac{1}{20}\int_{10}^{30} R(t)\,dt$ is avg rate of leak over $10 \le$ time ≤ 30 seconds =

c) $R'(25)=\frac{R(30)-R(20)}{30-20}=\frac{15-30}{10}=-1.5$ gal/min^2

$$\frac{1}{20}\int_{10}^{30} R(t)\,dt=\frac{1}{2}\left[10(30)+10(15)\right]$$

$= 22.5$ gal/min

d) $Q'(t)=0$

$t=18.805$ min

$Q''(t)=-16.78\sin(0.15t-1.25)(.15)^2$

$Q''(18.805)<0$ thus leak is at max at $t=18.805$ min