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AP CALCULUS BC **Problem Set Unit 4** **Name:** KEY

C

1. The asymptotes of the graph of the parametric equations $x=\frac{1}{t}$, $y=\frac{t}{t+1}$ are

(A) $x=0,\ y=0$ (B) $x=0$ only (C) $x=-1,\ y=0$
(D) $x=-1$ only (E) $x=0,\ y=1$

D

2. The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi}\sqrt{3+\cos\theta}\,d\theta$ (B) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$ (C) $2\int_0^{\pi/2}(3+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ (E) $2\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$

E

3. Which of the following integrals gives the length of the graph of $y=\tan x$ between $x=a$ and $x=b$, where $0<a<b<\frac{\pi}{2}$?

(A) $\int_a^b\sqrt{x^2+\tan^2 x}\,dx$

(B) $\int_a^b\sqrt{x+\tan x}\,dx$

(C) $\int_a^b\sqrt{1+\sec^2 x}\,dx$

(D) $\int_a^b\sqrt{1+\tan^2 x}\,dx$

(E) $\int_a^b\sqrt{1+\sec^4 x}\,dx$

A

4. The length of the curve $y=\ln\sec x$ from $x=0$ to $x=b$, where $0<b<\frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b\sec x\,dx$

(B) $\int_0^b\sec^2 x\,dx$

(C) $\int_0^b(\sec x\tan x)\,dx$

(D) $\int_0^b\sqrt{1+(\ln\sec x)^2}\,dx$

(E) $\int_0^b\sqrt{1+(\sec^2 x\tan^2 x)}\,dx$

A

5. If $x=t^2-1$ and $y=2e^t$, then $\frac{dy}{dx}=$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

C

6. The area of the region enclosed by the polar curve $r=1-\cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

D

7. A particle moves in the xy-plane so that at any time t its coordinates are $x=t^2-1$ and $y=t^4-2t^3$. At $t=1$, its acceleration vector is

(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$

D

8. The area of the region enclosed by the polar curve $r=\sin(2\theta)$ for $0\le\theta\le\frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

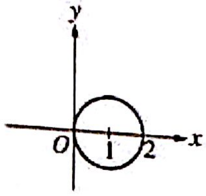
B

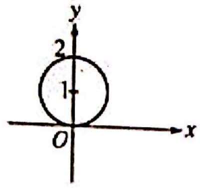
9. If $x=t^3-t$ and $y=\sqrt{3t+1}$, then $\frac{dy}{dx}$ at $t=1$ is

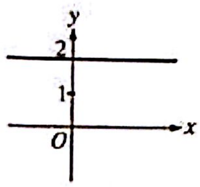
(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{8}{3}$ (E) 8

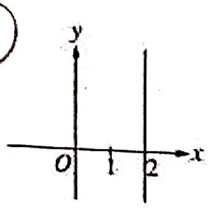
10. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

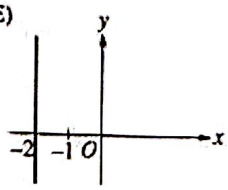
D

(A)


(B)


(C)


(D)


(E)


11. If $x = t^2 + 1$ and $y = t^3$, then $\dfrac{d^2y}{dx^2} =$

A

(A) $\dfrac{3}{4t}$ (B) $\dfrac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\dfrac{3}{2}$

12. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

D

(A) $\int_0^4 \sqrt{4t+1}\,dt$

(B) $2\int_0^4 \sqrt{t^2+1}\,dt$

(C) $\int_0^4 \sqrt{2t^2+1}\,dt$

(D) $\int_0^4 \sqrt{4t^2+1}\,dt$

(E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$

13. Consider the curve in the xy-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \ge 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

D

(A) 20.086 (B) 0.342 (C) −0.005 (D) −0.011 (E) −0.033

14. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2+2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

A

(A) $\left(\dfrac{3}{4}, 8\right)$ (B) $\left(\dfrac{3}{4}, 4\right)$ (C) $\left(\dfrac{1}{8}, 8\right)$ (D) $\left(\dfrac{1}{8}, 4\right)$ (E) $\left(-\dfrac{5}{16}, 4\right)$

15. If $x = e^{2t}$ and $y = \sin(2t)$, then $\dfrac{dy}{dx} =$

E

(A) $4e^{2t}\cos(2t)$ (B) $\dfrac{e^{2t}}{\cos(2t)}$ (C) $\dfrac{\sin(2t)}{2e^{2t}}$ (D) $\dfrac{\cos(2t)}{2e^{2t}}$ (E) $\dfrac{\cos(2t)}{e^{2t}}$

16. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \dfrac{\pi}{2}$, is given by

D

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\,dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t\,\sin t + 3\sin^2 t\,\cos t}\,dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\,dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t\,\sin^2 t + 9\sin^4 t\,\cos^2 t}\,dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\,dt$

~~Unit~~ Prob Set 4

1) $x = \frac{1}{t} = t^{-1}$

$y = \dfrac{t}{t+1}$

$$\frac{dy}{dx} = \frac{\frac{(t+1)-t}{(t+1)^2}}{-\frac{1}{t^2}}$$

$$\frac{dy}{dx} = \frac{1}{(t+1)^2}\cdot -t^2 = \frac{-t^2}{(t+1)^2}$$

VA: $t=-1$

$x=-1$

HA: $t=0$

$y = \frac{0}{0+1} = 0$

2) $$2\cdot\frac{1}{2}\int_0^{\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta$$

3) $$\int_a^b \sqrt{1+(\sec^2 x)^2}\,dx = \int_a^b \sqrt{1+\sec^4 x}\,dx$$

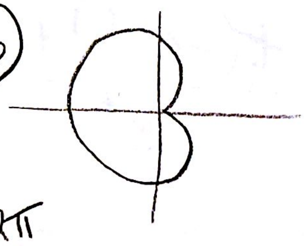
4) $$\int_0^b \sqrt{1+\left(\frac{1}{\cancel{\sec x}}\cdot\cancel{\sec x}\tan x\right)^2}\,dx$$

$\sqrt{1+\tan^2 x}$

$\sqrt{\sec^2 x}$

$\sec x$

5) $$\frac{\cancel{2}e^t}{\cancel{2}t} = \frac{e^t}{t}$$

6) 

$$\frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2 d\theta$$

$$= \frac{3}{2}\pi$$

7) $\langle 2t,\ 4t^3-6t^2\rangle$

$\langle 2,\ 12t^2-12t\rangle$

$\langle 2, 0\rangle$

8) $\frac{1}{2}\int_0^{\pi/2} (\sin(2\theta))^2 \, d\theta = \frac{\pi}{8}$

9) $\frac{1}{2}(3t+1)^{-1/2} \cdot 3 = \left.\dfrac{\frac{3}{2\sqrt{3t+1}}}{3t^2-1}\right|_{t=1} \quad \dfrac{\frac{3}{2\cdot 2}}{2} = \dfrac{\frac{3}{4}}{2} = \frac{3}{8}$

10) D - calc

11) $\dfrac{3t^2}{2t} = \dfrac{3t}{2} = \frac{3}{2}t \quad \dfrac{\frac{3}{2}}{2t} = \frac{3}{2} \cdot \frac{1}{2t}$
$= \frac{3}{4t}$

12) $\int_0^4 \sqrt{(2t)^2 + 1^2} \, dt$

13) $\dfrac{dy}{dx} = \dfrac{t(-e^{-t}) + e^{-t}}{e^t}$

$x = e^t$
$3 = e^t$
$\ln 3 = \cancel{\ln} e^t$
$t = \ln 3$

$\dfrac{(\ln 3)(-e^{-\ln 3}) + e^{-\ln 3}}{e^{\ln 3}} = -.011$

14) $(\ln(t^2+2t), 2t^2)$

$\left< \frac{1}{t^2+2t} \cdot 2t+2, 4t \right> \rightarrow \left< \frac{1}{4+4} \cdot 6, 8 \right>$

$\left< \frac{6}{8}, 8 \right>$

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Uni

$x = e^{2t}$ $y = \sin(2t)$

$\frac{dy}{dx} = \frac{\cancel{2}\cos(2t)}{\cancel{2}e^{2t}} = \frac{\cos(2t)}{e^{2t}}$

16) $\int_0^{\pi/2} \sqrt{}$

D

$(\cos t)^3$

$[3(\cos t)^2 \cdot -\sin t]^2$

$9\cos^4 t \sin^2 t$