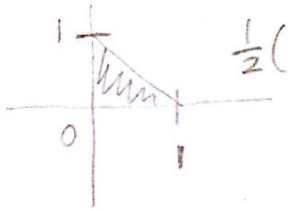
U4 Prob Set #2:

1) $\int_0^1 \sqrt{x^2-2x+1}\,dx = \int_0^1 \sqrt{(x-1)^2}\,dx = \int_0^1 |x-1|\,dx = \frac{1}{2}$

C



$\frac{1}{2}(1)(1)$

2) $\int \frac{x^2}{e^{x^3}}dx$ $\quad u = x^3$, $du = 3x^2dx$, $\frac{1}{3}du = x^2dx$

C

$$\frac{1}{3}\int \frac{1}{e^u}\,du = \frac{1}{3}\int e^{-u}\,du = \frac{-1}{3}\cdot\frac{1}{e^{x^3}} + C$$

3) Try $n=2$:

E

$$\int_0^1 x^2dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} - 0$$

$$\int_0^1 (1-x)^n dx = \int_0^1 (1-2x+x^2)\,dx = x - x^2 + \frac{x^3}{3}\Big|_0^1$$

$$= \left(1-1+\frac{1}{3}\right) - 0$$

$$= \frac{1}{3}$$

4) $\int_{-1}^{2} f(x)dx + \int_{2}^{3} f(x)dx$

D

$$= \int_{-1}^{2}(8-x^2)\,dx + \int_2^3 x^2dx = 8x - \frac{x^3}{3}\Big|_{-1}^{2} + \frac{x^3}{3}\Big|_2^3$$

$$= \left(16-\frac{8}{3}\right) - \left(-8+\frac{1}{3}\right) + \left(\frac{27}{3}-\frac{8}{3}\right)$$

$$\frac{40}{3} + \frac{23}{3} + \frac{19}{3} = \frac{82}{3} = 27\frac{1}{3}$$

5) $\int \sin(2x+3)\,dx$ $\quad u = 2x+3$, $du = 2dx$, $\frac{1}{2}du = dx$

D

$$\frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C$$
$$= -\frac{1}{2}\cos(2x+3) + C$$

6) $F(x) = \int_a^b f(x)$ $\quad F'(x) = f(x)$

D

$\hookrightarrow F(b) - F(a)$

7) $\int_0^1 (x+1)e^{x^2+2x}\,dx$ $\quad u = x^2+2x$, $du = (2x+2)dx$, $du = 2(x+1)dx$, $\frac{1}{2}du = (x+1)dx$, $u(0) = 0$, $u(1) = 3$

B

$$\frac{1}{2}\int_0^3 e^u\,du = \frac{1}{2}e^u\Big|_0^3$$
$$= \frac{1}{2}e^3 - \frac{1}{2}e^0 = \frac{1}{2}(e^3 - 1)$$

8) $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$, $du = -2x\,dx$, $-du = 2x\,dx$, $u(0) = 1$, $u\left(\frac{1}{2}\right) = \frac{3}{4}$

E

$$-\int_1^{3/4} u^{-1/2}\,du = \int_{3/4}^1 u^{-1/2}\,du = 2u^{1/2}\Big|_{3/4}^1$$

$$= 2\left(\sqrt{1} - \sqrt{\tfrac{3}{4}}\right) = 2\left(1 - \tfrac{\sqrt{3}}{2}\right)$$

$$2 - \sqrt{3}$$

9) $\int_{-1}^{1} f(x)dx = \int_{-1}^{0}(x+1)dx + \int_{0}^{1}\cos(\pi x)dx$

D

$u = \pi x$
$du = \pi dx$
$\frac{1}{\pi}du = dx$
$u(0) = 0$
$u(1) = \pi$

$\frac{x^2}{2}+x\Big|_{-1}^{0} + \frac{1}{\pi}\int_{0}^{\pi}\cos u\,du$

$0-(\frac{1}{2}-1) + \frac{1}{\pi}(\sin\pi - \sin 0)$

$\frac{1}{2} + \frac{1}{\pi}(0)$

$\frac{1}{2}$

10) $\frac{1}{2}(\frac{1}{3})(1+2(\frac{16}{9})+2(\frac{25}{9})+4)$

D

$\frac{1}{6}(1+\frac{32}{9}+\frac{50}{9}+4)$

$\frac{1}{6}(\frac{9}{9}+\frac{32}{9}+\frac{50}{9}+\frac{36}{9})$

$\frac{1}{6}(\frac{127}{9}) = \frac{127}{54}$

11) $\int_{1}^{2} x^{-3}dx = \frac{x^{-2}}{-2}\Big|_{1}^{2} = \frac{2^{-2}}{-2} - \frac{1^{-2}}{-2}$

D

$= -\frac{1}{8} - (-\frac{1}{2})$

$= -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$

12) $\frac{dy}{dx} = \cos(2x)$

C

$\int dy = \int\cos(2x)dx$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$

$y = \frac{1}{2}\int\cos u\,du$

$= \frac{1}{2}\sin(2x)+C$

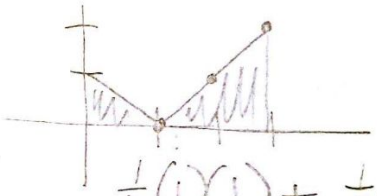
13) $\int_{-1}^{1} e^{-x^2}dx = K$ $\int_{-1}^{0} e^{-x^2}dx = \frac{1}{2}K$

D

even function

14) $\int_{0}^{3} |x-1|dx$

D



$\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2)$

$\frac{1}{2} + 2$

$\frac{5}{2}$

15)

C

16) $\frac{d}{dx}\int_{2}^{x} \sqrt{1+t^2}\,dt = \sqrt{1+x^2}$

C