Name: Key

Unit 4 Five 'n' One

Points Earned	~~14~~	13	12/11	10/9	8/7	6	5	4	3/2/1/0
Grade	100	94	89	84	79	74	69	60	45

Place answers here!

Calculator Inactive

FORM A

1. B

Let f be a differentiable function for all x. Which of the following must be true?

I. $\int_0^3 f(x)dx = f(3) - f(0)$ $F(3)-F(0)$

II. $\int_3^x f'(x)dx = f(x)$ $f(x)-f(3)$

III. $\frac{d}{dx}\int_3^x f(x)dx = f(x)$

FTC Pt 2

a. II only
b. III only
c. I and II only
d. II and III only
e. I, II, and III

2. B

Let $r(x)$ be a positive, strictly increasing function that is concave down on the interval (3,6). List the following values in order from least to greatest.

I. Right Riemann sum approximation of $\int_3^6 r(x)dx$ with 3 subdivisions of equal length over

II. Left Riemann sum approximation of $\int_3^6 r(x)dx$ with 3 subdivisions of equal length under

III. Trapezoidal sum approximation of $\int_3^6 r(x)dx$ with 3 subdivisions of equal length under

IV. $\int_3^6 r(x)dx$ actual

~~a.~~ I < III < IV < II
b. II < III < IV < I
~~c.~~ I < IV < III < II
d. II < IV < III < I
~~e.~~ III < II < IV < I

3. A

$\int_1^e \ln x dx=$

a. $\lim_{n\to\infty} \sum_{i=1}^{n} \ln(1+i\cdot\frac{e-1}{n})\cdot\frac{e-1}{n}$
~~b.~~ $\lim_{n\to\infty} \sum_{i=1}^{n} \ln(1+\frac{i}{n})\cdot\frac{e}{n}$
~~c.~~ $\lim_{n\to\infty} \sum_{i=1}^{n} \ln(1+\frac{ei}{n})\cdot\frac{e}{n}$
d. $\lim_{n\to\infty} \sum_{i=1}^{n} \ln(i\cdot\frac{e-1}{n})\cdot\frac{e-1}{n}$
~~e.~~ $\lim_{n\to\infty} \sum_{i=0}^{n} \ln(1+i\cdot\frac{e-1}{n})\cdot\frac{e-1}{n}$

W $\Delta x = \frac{b-a}{n} = \frac{e-1}{n}$

$x_k = a + k\Delta x = 1 + k\left(\frac{e-1}{n}\right)$

L $f(x_k) = \ln\left(1+k\left(\frac{e-1}{n}\right)\right)$

Area $= \ln\left(1+k\left(\frac{e-1}{n}\right)\right)\cdot\left(\frac{e-1}{n}\right)$

4. D

Let f be a continuous function such that $\int_2^3 f(2x)dx = 8$. What is the value of $\int_4^6 f(x)dx$?

a. 4
b. 8
c. 12
d. 16
e. 32

$u=2x$ $u(2)=4$
$du=2dx$ $u(3)=6$
$\frac{1}{2}du=dx$

$\int_2^3 f(2x)dx = \frac{1}{2}\int_4^6 f(u)du$

$8=\frac{1}{2}A$ $A=16$

5. D

Let $g(x) = \int_1^x f(t)dt$. The table below gives selected values of f', the derivative of f, on the interval $1 \le x \le 7$. If f' is continuous and has only the three zeros as shown in the table, for which values of x does g have a point of inflection?

x	1	2	3	4	5	6	7
$f'(x)$	5	0	−3	0	−2	0	4

a. 2 only
b. 4 only
c. 6 only
d. 2 and 6 only
e. 2, 4, and 6

$g(x)=\int_1^x f(t)dt$ $g'(x)=f(x)$

$g''(x)=f'(x)$

$f'(x)=0$ and changes signs

e: ______________

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Calculator ACTIVE   AFTERNOON

A baseball team held a contest during its last home game of the season to name its newly designed mascot. Baseball fans placed nominations into a box between noon ($t = 0$) and 8 pm ($t = 8$). The number of nominations in the box t hours after noon is modeled by a differentiable function M for $0 \le t \le 8$. Values of $M(t)$, in hundreds of nominations, at various times t are shown in the table below.

t (hours)	0	2	5	7	8
$M(t)$ (hundreds of nominations)	0	4	13	21	23

a) Use the data in the table to approximate the rate, in hundreds of nominations per hour, at which nominations were being deposited at 6 pm. Show the computations that lead to your answer.

$M'(6) \approx \frac{M(7)-M(5)}{7-5} = \frac{21-13}{2} = 4$ hundred nominations per hour

1 answer

b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 M(t)dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 M(t)dt$ in terms of the number of nominations.

$\frac{1}{8}\int_0^8 M(t)dt \approx \frac{1}{2}\cdot\frac{1}{8}\left[2(0+4)+3(4+13)+2(13+21)+1(21+23)\right]$

$\frac{1}{2}\cdot\frac{1}{8}(8+51+68+44)$

$= 10.688$ or 10.687

avg # of hundreds of nominations in box between noon and 8 pm

1 TAM
1 Approximation
1 explain

c) At 8 pm, volunteers began to process the nominations. They processed the nominations at a rate modeled by the function B, where $B(t) = t^3 - 30t^2 + 298t - 976$ hundreds of nominations per hour $8 \le t \le 12$. According to the model, how many nominations had not been processed by midnight?

$23 - \int_8^{12} B(t)dt = 23 - 16 = 7$ hundred entries

1 integral
1 answer

d) According to the model from part (c), at what time were the nominations being processed most quickly? Justify your answer.

$B'(t) = 0$ $t = 9.184$ (A) $t = 10.816$ (B)

t	$B(t)$
8	0
A	5.089
B	2.911
12	8

Nominations processed most quickly at time $t=12$.

1 $B'(t)=0$
1 Identifies candidates
1 Answer with justification

)

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Calculator ACTIVE

MORNING

There were no leaves in Barney's backyard when the wind started to blow at 6 am. From 6am to 3 pm, leaves piled up in the backyard at a rate modeled by $b(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since 6 am. Barney starts raking leaves at noon ($t = 6$). The rate $c(t)$, in cubic feet per hour, at which Barney rakes leaves from the backyard at time t hours after 6 am is modeled by

$$c(t) = \begin{cases} 0 \; for \; 0 \le t < 6 \\ 125 \; for \; 6 \le t < 7 \\ 108 \; for \; 7 \le t \le 9 \end{cases}$$

a) How many cubic feet of leaves have accumulated on the driveway by noon?

$$\int_0^6 b(t)\,dt = 142.275 \quad ft^3$$
$$(142.274)$$

b) Find the rate of change of the volume of leaves on the driveway at 2:30 pm.

$$b(8.5) - c(8.5) = -75.411 \quad ft^3/hr$$

c) Let $d(t)$ represent the total amount of leaves, in cubic feet, that Barney has removed from the backyard at time t hours after 6 am. Express d as a piecewise-defined function with domain $0 \le t \le 9$.

$d(0)=0 \qquad 0<t\le 6,\ d(t) = d(0) + \int_0^t c(s)\,ds = 0 + \int_0^t 0\,ds = 0$

$6<t\le 7,\ d(t) = d(6) + \int_6^t c(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6)$

$7<t\le 9,\ d(t) = d(7) + \int_7^t c(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7)$

d) How many cubic feet of leaves are in the backyard at 3 pm?

$$d(t) = \begin{cases} 0, & 0<t\le 6 \\ 125(t-6), & 6<t\le 7 \\ 125+108(t-7), & 7<t\le 9 \end{cases}$$

Amt of Leaves:

$$\left(\int_0^9 b(t)\,dt\right) - d(9) =$$

$$26.335 \text{ or } 26.334 \quad ft^3$$