me: ____________________

Unit 4 Five 'n' One AM

Points Earned	14	13	12/11	10/9	8/7	6	5	4	3/2/1/0
Grade	100	94	89	84	79	74	69	60	45

Place answers here!

Calculator INactive

1. C

If $F(x) = \int_0^{x^2} \sqrt{t+3}\,dt$, what is $F'(x)$?

a. $\sqrt{x^2+3}$
b. $\frac{1}{2\sqrt{x^2+3}}$
c. $2x(\sqrt{x^2+3})$
d. $\frac{2(x^2+3)^{3/2}}{3}$
e. None of the above

$\sqrt{x^2+3} \cdot 2x$

2. C

Let R be the region between the function $f(x) = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 4$. Using the Trapezoidal Rule, compute the area when there are four equal subdivisions.

a. 52
b. 35
c. 26
d. 44
e. None of these

0	1	2	3	4
1	2	5	10	17

$\frac{35}{17} \frac{1}{2}$

$\frac{1}{2}(1)(1+2(2)+2(5)+2(10)+17)$

$\frac{1}{2}(52) = 26$

3. B

What is $f(x)$ if $f'(x) = \frac{2x}{x^2-1}$ and $f(2) = 0$?

a. $f(x) = \ln|x^2-1|$
b. $f(x) = \ln|x^2-1| - \ln 3$
c. $f(x) = \ln|x^2-1| + \ln 3$
d. $f(x) = 2\ln x - x^2$
e. $f(x) = 2\ln x - x^2 - 2\ln 2 + 4$

$f(x) = f(2) + \int_2^x \frac{2x}{x^2-1}\,dx$

$u = x^2-1$
$du = 2x\,dx$
$x = \sqrt{u+1}$
$u(2) = 3$

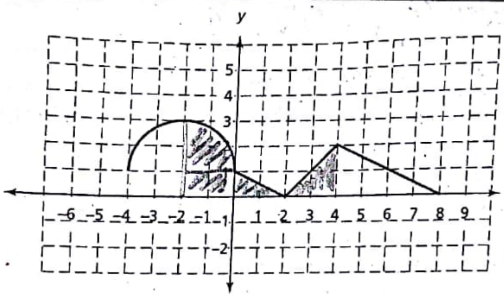
$f(x) = 0 + \int \frac{1}{u}\,du = \ln|u|$

$\ln|x^2-1| \Big|_2^x$

$\ln|x^2-1| - \ln 3$

4. D

$\frac{1}{4}\pi(2)^2 = \pi$
$2 \cdot 1 = 2$
$\frac{1}{2}(2)(1) = 1$
$\frac{1}{2}(2)(2) = 2$



The graph of $f(x)$ shown consists of three line segments and one semicircle. Let $g(x) = \int_{-2}^{x} f(t)\,dt$. Which of the following statements must be false?

a. $g(4) = \int_{-2}^{4} f(t)\,dt = \pi + 5$
b. $g(-4) = \int_{-2}^{-4} f(t)\,dt = -\pi - 2$ $\quad -\int_{-4}^{-2} f(t)\,dt$
c. $g'(6) = 1$ $\quad f(x)\quad f(6)$
d. $g(x)$ has a relative maximum at $x = 4$ $\quad f(x)$ pos to neg changes sign
e. $g(x)$ has a point of inflection at $x = 2$ $\quad f'(x)$

5. B

$-(\pi+2)$
$-\pi-2$

If $R(x)$ is an even function and $S(x)$ is an odd function where $\int_{-a}^{0} R(x)\,dx = 5$ and $\int_0^a S(x)\,dx = -3$, find the value of $\int_{-a}^{a}[2(S(x)) - R(x) + 3]\,dx$.

a. $-10 + 3a$
b. $-10 + 6a$
c. $10 + 6a$
d. $-6 - 6a$
e. $-6 + 6a$

$2\int_{-a}^{a} S(x)\,dx - \int_{-a}^{a} R(x)\,dx + \int_{-a}^{a} 3\,dx$

$2 \cdot 0 - 10 + 3x\Big|_{-a}^{a}$

$-10 + 3a - (-3a)$

$6a - 10$

$-10 + 6a$

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Unit 4 Five 'n' One

Calculator ACTIVE – one point per blank

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected time t for the first 20 minutes are given in the table above.

a) Use the data in the table to estimate $W'(12)$. Show the computation that leads to your answer. Use correct units.

$$\frac{W(15)-W(9)}{15-9} = \frac{67.9-61.8}{15-9}$$

(or 1.016)

$W'(12) =$ 1.017 °F/min = units

b) Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Write your answer with correct units.

$$\int_0^{20} W'(t)dt = W(20) - W(0) = 71-55$$

$\int_0^{20} W'(t)dt =$ 16 °F = units

c) For $0 \le t \le 20$, the average temperature of water in the tub is $\frac{1}{20}\int_0^{20} W(t)dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\frac{1}{20}\int_0^{20} W(t)dt = \frac{4(55)+5(57.1)+6(61.8)+5(67.9)}{20}$$

LRAM = 1215.8 $\frac{1}{20}\int_0^{20} W(t)dt =$ 60.79 over/underestimate? under

Why? W(t) is increasing

d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$W(25) = W(20) + \int_{20}^{25} W'(t)dt$$

71

Answer = 73.043