Review Part I

AP Calculus BC
Chapter 9
MC Practice 9.7-9.10

Name______________________

Date______________ Period________

NO CALCULATORS for #1–5

1. The power series $1+2x+4x^2+8x^3+\cdots+2^{n-1}x^{n-1}+\cdots$ converges for what values of x?

(A) $x=0$ only

(B) $-\frac{1}{2}<x<\frac{1}{2}$ only

(C) $-1<x<1$ only

(D) $-2<x<2$ only

(E) All real numbers

2. The Taylor series for $\frac{\sin(x^2)}{x^2}$ centered at $x=0$ is

(A) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$

(B) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{(2k+1)!}$

(C) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k)!}$

(D) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{4k}}{(2k+1)!}$

(E) $\frac{1}{x}+\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k-1}}{(2k-1)!}$

3. The first three nonzero terms in the Maclaurin series of xe^{-x} are

(A) $x-x^2-\frac{x^3}{2!}$

(B) $x-x^2+\frac{x^3}{2!}$

(C) $-x+x^2-\frac{x^3}{2!}$

(D) $x+x^2+\frac{x^3}{2!}$

(E) $1-x+\frac{x^2}{2!}$

4. The Taylor Series of a function $f(x)$ about $x=3$ is given by

$$f(x)=1+\frac{3(x-3)}{1!}+\frac{5(x-3)^2}{2!}+\frac{7(x-3)^3}{3!}+\cdots+\frac{(2n+1)(x-3)^n}{n!}+\cdots$$

What is the value of $f'''(3)$?

(A) 0

(B) $1.\overline{6}$

(C) 2.5

(D) 5

(E) 7

5. Let $f(x)$ be a function that is continuous and differentiable for all x. The derivative of this function is given by the power series

$$f'(x)=3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$$

If $f(0)=2$, then $f(x)=$

(A) $0+3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$

(B) $2+3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$

(C) $\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

(D) $2-\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

(E) $2+\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

(calculator allowed)

6. The power series $\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\cdots$ converges for all real numbers. For values in the interval $\left[0,\frac{\pi}{2}\right]$, what is the minimum number of terms of the power series necessary to approximate the value of $\cos(x)$ with an error whose absolute value is less than 0.0001?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

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Chapter 9
Review 9.7–9.10

Name________________________________

Date________________________ Period________

Use the definition of a Taylor Polynomial,

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

to find the Taylor Polynomials of degree n centered at c for the following functions:

1. $y = \sqrt{x}$, $n = 3$, $c = 1$

2. $y = 2^x$, $n = 4$, $c = -1$

3. Find the interval of convergence for: $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)}$

4. Find the interval of convergence for $f(x)$, $f'(x)$, and $\int f(x)\,dx$.

(a) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x+3)^{n+1}}{n}$

(b) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{n}}{4^{n}n}$

5. Find the power series centered at $c = -1$ for the function: $f(x) = \dfrac{2}{x+3}$

Find the interval of convergence for the power series.

Find the Maclaurin series for the following functions using power series for elementary functions. List the first four non-zero terms and the general term.

6. $y = \cos \frac{x}{2}$

7. $y = x \sin x^2$

8. $y = e^{-2x^2}$

9. Use a power series to approximate the following definite integral with an error less than 0.001.

$$\int_0^1 \sin x^2 \, dx$$

AP® CALCULUS BC
2004

Question 6

Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

(b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

(c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$.

(d) Let G be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for G about $x = 0$.

AP® CALCULUS BC
2003

Question 6

The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

(c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

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Question 6

The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

(a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

(b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

AP® CALCULUS BC

2002 (Form B)

Question 6

The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \le x < 1$.

(a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.

(b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

(c) Give a value of p such that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.

(d) Give a value of p such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.

AP Calculus BC
Chapter 9
MC Practice 9.7-9.10

Name Answer Key

Date ____________ Period ________

NO CALCULATORS for #1–5

1. The power series $1+2x+4x^2+8x^3+\cdots+2^{n-1}x^{n-1}+\cdots$ converges for what values of x?

geometric $r=2x$

$|2x|<1$

$|x|<\frac{1}{2}$

(A) $x=0$ only

B (B) $-\frac{1}{2}<x<\frac{1}{2}$ only

(C) $-1<x<1$ only

(D) $-2<x<2$ only

(E) All real numbers

2. The Taylor series for $\frac{\sin(x^2)}{x^2}$ centered at $x=0$ is

$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin x^2 = \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!}$

$\frac{\sin x^2}{x} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n}}{(2n+1)!}$

(A) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$

(B) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k}}{(2k+1)!}$

(C) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2k)!}$

D (D) $\sum_{k=0}^{\infty}\frac{(-1)^k x^{4k}}{(2k+1)!}$

(E) $\frac{1}{x}+\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k-1}}{(2k-1)!}$

3. The first three nonzero terms in the Maclaurin series of xe^{-x} are

$e^x = 1+x+\frac{x^2}{2!}$

$e^{-x} = 1-x+\frac{x^2}{2!}$

$xe^{-x} = x-x^2+\frac{x^3}{2!}$

(A) $x-x^2-\frac{x^3}{2!}$

B (B) $x-x^2+\frac{x^3}{2!}$

(C) $-x+x^2-\frac{x^3}{2!}$

(D) $x+x^2+\frac{x^3}{2!}$

(E) $1-x+\frac{x^2}{2!}$

center

4. The Taylor Series of a function $f(x)$ about $x=3$ is given by

$$f(x)=1+\frac{3(x-3)}{1!}+\frac{5(x-3)^2}{2!}+\frac{7(x-3)^3}{3!}+\cdots+\frac{(2n+1)(x-3)^n}{n!}+\cdots$$

What is the value of $f'''(3)$?

(A) 0

(B) $1.\overline{6}$

(C) 2.5

(D) 5

(E) 7

$n=3$: $\frac{f'''(3)}{3!}(x-3)^3$

E

5. Let $f(x)$ be a function that is continuous and differentiable for all x. The derivative of this function is given by the power series

$$f'(x)=3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$$

If $f(0)=2$ then $f(x)=$

(A) $0+3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$

(B) $2+3x-\frac{9x^3}{2}+\frac{81x^5}{40}-\frac{3x^7}{60}+\cdots$

(C) $\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

(D) $2-\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

(E) $2+\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{27x^6}{80}-\frac{3x^8}{480}+\cdots$

$\int f'(x)dx=\frac{3x^2}{2}-\frac{9x^4}{8}+\frac{81x^6}{4\cdot 6}-\frac{3x^8}{480}+\cdots+C$

E

(calculator allowed)

6. The power series $\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\cdots$ converges for all real numbers. For values in the interval $\left[0,\frac{\pi}{2}\right]$, what is the minimum number of terms of the power series necessary to approximate the value of $\cos(x)$ with an error whose absolute value is less than 0.0001?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

B

Lagrange: max $\left|\frac{f^{(n)}(z)}{n!}x^n\right|$ on interval $\left[0,\frac{\pi}{2}\right]$

max value of $|f^{(n)}(z)|=1$ & max value of $x=\frac{\pi}{2}$

$\left|\frac{\left(\frac{\pi}{2}\right)^n}{n!}\right| \Longrightarrow n=10$ $\left[\frac{\left(\frac{\pi}{2}\right)^{10}}{10!}<.0001\right]$

$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}$

AP Calculus BC
Chapter 9
Review 9.7–9.10

MULTIPLE CHOICE ANSWERS
1. B   2. D   3. B
4. E   5. E   6. B

Name ORSI

Date________ Period________

Use the definition of a Taylor Polynomial,

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots + \boxed{\frac{f^{(n)}(c)}{n!}(x-c)^n}$$

✗ Taylor term

to find the Taylor Polynomials of degree n centered at c for the following functions:

1. $y = \sqrt{x}$, $n = 3$, $c = 1$

$n=0$ $\quad f(x) = \sqrt{x}$ $\quad f(1) = 1$

$n=1$ $\quad f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(1) = \frac{1}{2}$

$n=2$ $\quad f''(x) = -\frac{1}{4}x^{-3/2}$ $\quad f''(1) = -\frac{1}{4}$

$n=3$ $\quad f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(1) = \frac{3}{8}$

$$T_3(x) = \frac{1(x-1)^0}{0!} + \frac{\frac{1}{2}(x-1)^1}{1!} + \frac{-\frac{1}{4}(x-1)^2}{2!} + \frac{\frac{3}{8}(x-1)^3}{3!}$$

$$= \boxed{1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{4\cdot 2!} + \frac{3(x-1)^3}{8\cdot 3!}}$$

2. $y = 2^x$, $n = 4$, $c = -1$

$n=0$ $\quad f(x) = 2^x$ $\quad f(-1) = \frac{1}{2}$

$n=1$ $\quad f'(x) = 2^x \ln 2$ $\quad f'(-1) = \frac{\ln 2}{2}$

$n=2$ $\quad f''(x) = 2^x(\ln 2)^2$ $\quad f''(-1) = \frac{(\ln 2)^2}{2}$

$n=3$ $\quad f'''(x) = 2^x(\ln 2)^3$ $\quad f'''(-1) = \frac{(\ln 2)^3}{2}$

$n=4$ $\quad f^{(4)}(x) = 2^x(\ln 2)^4$ $\quad f^{(4)}(-1) = \frac{(\ln)^4}{2}$

$$T_4(x) = \frac{\frac{1}{2}(x+1)^0}{0!} + \frac{\frac{\ln 2}{2}(x+1)^1}{1!} + \frac{\frac{(\ln 2)^2}{2}(x+1)^2}{2!} + \frac{\frac{(\ln 2)^3}{2}(x+1)^3}{3!} + \frac{\frac{(\ln 2)^4}{2}(x+1)^4}{4!}$$

$$= \boxed{\frac{1}{2} + \frac{(\ln 2)(x+1)}{2} + \frac{(\ln 2)^2(x+1)^2}{2\cdot 2!} + \frac{(\ln 2)^3(x+1)^3}{2\cdot 3!} + \frac{(\ln 2)^4(x+1)^4}{2\cdot 4!}}$$

3. Find the interval of convergence for: $\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)}$ $(c=0)$

R.O.C. (Ratio Test)

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{2^{n+1}(n+2)} \cdot \frac{2^n(n+1)}{x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{x(n+1)}{2(n+2)}\right| = \left|\frac{x}{2}\right| < 1$$

$$|x| < 2$$

R.O.C = 2

TEST ENDPOINTS:

$x=-2$: $\sum_{n=0}^{\infty}\frac{(-2)^n}{2^n(n+1)} = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{2^n(n+1)} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$

alternating harmonic converges
∴ include $x=-2$

$x=2$: $\sum_{n=0}^{\infty}\frac{2^n}{2^n(n+1)} = \sum_{n=0}^{\infty}\frac{1}{n+1}$ strictly harmonic diverges

∴ exclude $x=2$

$\boxed{\text{I.O.C.} \quad [-2, 2)}$

SHOW SERIES

4. Find the interval of convergence for $f(x)$, $f'(x)$, and $\int f(x)\,dx$.

(a) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x+3)^{n+1}}{n}$ $(c=-3)$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x+3)^{n+2}}{(n+1)}\cdot\frac{n}{(-1)^{n+1}(x+3)^{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x+3)\,n}{(n+1)}\right| = |x+3|<1$$

R.O.C = 1

$x=-4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^{n+1}}{n} = \sum_{n=1}^{\infty}\frac{1}{n}$ diverges

$x=-2$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(1)^{n+1}}{n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ converges

IOC: $(-4,-2]$

$f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+1)(x+3)^{n}}{n}$

check $x=-2$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n+1)\cdot(1)^n}{n}$

$\lim_{n\to\infty}\frac{n+1}{n}=1\neq 0$ diverges

$f'(x)$ I.O.C: $(-4,-2)$

$\int f(x)dx = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x+3)^{n+2}}{n(n+2)}$

check $x=-4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^{n+2}}{n(n+2)} = \sum_{n=1}^{\infty}\frac{(-1)}{n^2+2n}$

converges by limit comparison test to $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\int f(x)dx$ I.O.C. : $[-4,-2]$

(b) $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{n}}{4^{n}n}$ $(c=0)$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}x^{n+1}}{4^{n+1}(n+1)}\cdot\frac{4^n n}{(-1)^{n+1}x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{x\cdot n}{4(n+1)}\right| = \left|\frac{x}{4}\right|<1$$

$|x|<4$

R.O.C: = 4

$x=-4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-4)^n}{4^n n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n 4^n}{4^n n}$

$= \sum_{n=1}^{\infty}\frac{-1}{n}$ strictly harmonic (diverges)

$x=4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^n}{4^n n}$ alternating harmonic (converges)

I.O.C. : $(-4,4]$

$f'(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot n x^{n-1}}{4^n\cdot n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{n-1}}{4^n} = f'(x)$

*no need to re-index!

check $x=4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^{n-1}}{4^n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}4^n 4^{-1}}{4^n}$

geometric $|r|=1$ diverge

$f'(x)$ IOC: $(-4,4)$

$\int f(x)dx = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{n+1}}{4^n n(n+1)}$

check $x=-4$: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-4)^{n+1}}{4^n n(n+1)} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n 4^n 4}{4^n n(n+1)}$

$\sum_{n=1}^{\infty}\frac{(-4)}{n(n+1)}$ converges by LCT to $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\int f(x)dx$ I.O.C $[-4,4]$

*need (x+1)

5. Find the power series centered at $c=-1$ for the function: $f(x)=\frac{2}{x+3} = \frac{2}{3+x} = \frac{2}{2+(x+1)} = \frac{1}{1+\frac{(x+1)}{2}}$

*re-write as geometric sum $\frac{a}{1-r}$, $a=1$, $r=\frac{-(x+1)}{2}$

Find the interval of convergence for the power series.

*can use ratio test or

GEOMETRIC SERIES TEST: Converges if $\left|\frac{-(x+1)}{2}\right| < 1$

$|x+1| < 2$

I.O.C: $(-3, 1)$

*no need to test endpoints (geometric will never include endpoints)

Find the Maclaurin series for the following functions using power series for elementary functions. List the first four non-zero terms and the general term.

6. $y=\cos\frac{x}{2}$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$$

$$\therefore \cos\left(\frac{x}{2}\right) = 1-\frac{\left(\frac{x}{2}\right)^2}{2!}+\frac{\left(\frac{x}{2}\right)^4}{4!}-\frac{\left(\frac{x}{2}\right)^6}{6!}+\cdots+\frac{(-1)^n\left(\frac{x}{2}\right)^{2n}}{(2n)!}+\cdots$$

$$= 1-\frac{x^2}{2^2\cdot 2!}+\frac{x^4}{2^4\cdot 4!}-\frac{x^6}{2^6 6!}+\cdots+\frac{(-1)^n x^{2n}}{2^{2n}(2n)!}+\cdots$$

7. $y=x\sin x^2$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots$$

$$\sin x^2 = (x^2)-\frac{(x^2)^3}{3!}+\frac{(x^2)^5}{5!}-\frac{(x^2)^7}{7!}+\cdots+\frac{(-1)^n(x^2)^{2n+1}}{(2n+1)!}+\cdots$$

$$x\sin x^2 = x^3-\frac{x^7}{3!}+\frac{x^{11}}{5!}-\frac{x^{15}}{7!}+\cdots+\frac{(-1)^n x^{4n+3}}{(2n+1)!}+\cdots$$

8. $y=e^{-2x^2}$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$$

$$e^{-2x^2} = 1+(-2x^2)+\frac{(-2x^2)^2}{2!}+\frac{(-2x^2)^3}{3!}+\cdots+\frac{(-2x^2)^n}{n!}+\cdots$$

$$= 1-2x^2+\frac{2^2x^4}{2!}-\frac{2^3x^6}{3!}+\cdots+\frac{(-1)^n 2^n x^{2n}}{n!}+\cdots$$

9. Use a power series to approximate the following definite integral with an error less than 0.001. $\left(\frac{1}{1000}\right)$

$$\int_0^1 \sin x^2\,dx \approx \int_0^1\left(x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}-\frac{x^{14}}{7!}+\cdots\right)dx$$

*don't know how many terms are necessary

$$= \left[\frac{x^3}{3}-\frac{x^7}{7\cdot 3!}+\frac{x^{11}}{11\cdot 5!}-\frac{x^{15}}{15\cdot 7!}+\cdots\right]_0^1$$

$$= \frac{1}{3}-\frac{1}{7\cdot 3!}+\frac{1}{11\cdot 5!}-\frac{1}{15\cdot 7!}+\cdots$$

$$= \left|\frac{1}{3}-\frac{1}{42}\right|+\frac{1}{1320}$$

*first term $< \frac{1}{1000}$

$$\therefore \int_0^1 \sin x^2 \approx \frac{1}{3}-\frac{1}{42} = \frac{13}{42}$$