Test Review Unit 2:

1) $\sum_{n=1}^{\infty} 5^{k-3} \cdot 9^k \qquad 5^k \cdot 5^{-3} \cdot 9^k = \frac{1}{125}(45)^k$ div by geo

C

2) I. $\sum \frac{1}{n+2\sqrt{n}}$ Compare to $\frac{1}{n}$ div

B

LCT: $\lim_{n\to\infty} \frac{1}{n+2\sqrt{n}} \cdot \frac{n}{1} = 1 \quad \therefore$ div

II. $\sum \frac{n}{\sqrt{n^5-1}}$ Compare to $\frac{n}{n^{5/2}} = \frac{1}{n^{3/2}}$ conv

LCT: $\lim_{n\to\infty} \frac{n}{\sqrt{n^5-1}} \cdot \frac{n^{5/2}}{n} = 1 \quad \therefore$ conv

III. $\sum \frac{\ln n}{n^2} \qquad \int_1^{\infty} \frac{\ln x}{x^2}\,dx \to \lim_{b\to\infty} \int_1^b \ln x \cdot x^{-2}\,dx$

$u = \ln x \quad dv = x^{-2}dx$

$du = \frac{1}{x}dx \quad v = \frac{-1}{x}$

$-\frac{1}{x}\ln x + \int \frac{1}{x} \cdot \frac{1}{x}\,dx = -\frac{\ln x}{x} - \frac{1}{x}\Big|_1^b$

$\lim_{b\to\infty} \left(-\underset{0}{\frac{\ln b}{b}} - \underset{0}{\frac{1}{b}}\right) - (0-1) = 1 \quad \therefore$ conv

3) I. $\sum \frac{1}{n^2}$ conv p-series

D

II. $\sum \frac{1}{n}$ div p-series

III. $\sum \frac{(-1)^n}{\sqrt{n}}$ conv AST $\quad \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ ✓

$\sqrt{n} \leq \sqrt{n+1}$

4) $f(x) = \ln(x+4+e^{-3x})$

B $f'(x) = \dfrac{1}{x+4+e^{-3x}} \cdot (1-3e^{-3x})$

$f'(0) = -\dfrac{2}{5}$

5) D - f(x) is inc where f'(x) is pos (above x-axis)

6) I. $\sum \dfrac{2}{k^2+1}$ compare to: $\dfrac{2}{k^2}$ big conv

A

DCT: conv

II. $\sum \left(\frac{6}{7}\right)^k$ conv by geo

III. $\sum \dfrac{(-1)^k}{k}$ conv by AST $\lim\limits_{k\to\infty} \dfrac{1}{k} = 0$ ✓

$\dfrac{1}{k+1} \le \dfrac{1}{k}$ ✓

$k \le k+1$

7) $\lim\limits_{x\to 1} \dfrac{\int_1^x e^{t^2}dt}{x^2-1}$

C

L'H: $\lim\limits_{x\to 1} \dfrac{e^{x^2}}{2x} = \dfrac{e}{2}$

8) $\displaystyle\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}dx$ $u = 9-x^2$ $du = -2x\,dx$ $\displaystyle\lim_{b\to\infty}\int_4^b \frac{-2x}{\sqrt[3]{9-x^2}}dx$

E

$\int u^{-1/3}du = \frac{3}{2}u^{2/3}$ $\displaystyle\lim_{b\to\infty} \frac{3}{2}(9-x^2)^{2/3}\Big|_4^b$

∴ divergent

$\displaystyle\lim_{b\to\infty} \frac{3}{2}(9-b^2)^{2/3} - \frac{3}{2}(9-4^2)^{2/3}$

9) $\int x^2 \cos(x^3)dx$ $\quad u = x^3$

C $\quad du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3}\int \cos u \, du$

$= \frac{1}{3} \sin x^3 + C$

10) I. $\sum \frac{(-1)^{n+1}}{1.1^n}$ $\quad \lim_{n\to\infty} \frac{1}{1.1^n} = 0$ $\quad \frac{1}{1.1^{n+1}} \le \frac{1}{1.1^n}$

$1.1^n \le 1.1^{n+1}$

A $\quad \therefore$ ABS conv $\quad \sum \frac{1}{1.1^n} \to \left(\frac{1}{1.1}\right)^n$ conv by geo

II. $\sum \frac{(-1)^n}{n\cos(\pi n)} \to \frac{1}{n}$ div

III. $\sum \frac{(-1)^{n+1}}{2^n + .5^n}$ $\quad \lim_{n\to\infty} \frac{1}{2^n + .5^n} = 0$ $\quad \frac{1}{2^{n+1} + .5^{n+1}} \le \frac{1}{2^n + .5^n}$

$\therefore$ ABS conv $\quad \sum \frac{1}{2^n + .5^n}$ compare to $\frac{1}{2^n} = \left(\frac{1}{2}\right)^n$

big conv by geo

11) $\frac{K}{4} < 1$ $\quad (-1)^{Kn}$ alternate

C $\quad K < 4$ $\quad K \to$ odd

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12) I. $\sum \frac{n}{n+2}$ n^{th} term div

C II. $\sum \frac{\cos(n\pi)}{n}$ AST Conv $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n+1} \leq \frac{1}{n}$ ✓

III. $\sum \frac{1}{n}$ div

13) I. $\sum \frac{1}{n^2}$ conv p-series

C

II. $\sum \frac{1}{n}$ div p-series

III. $\sum \frac{(-1)^{n+1}}{3^{n-1}}$ AST Conv $\lim_{n\to\infty} \frac{1}{3^{n-1}} = 0$ ✓

$\frac{1}{3^n} \leq \frac{1}{3^{n-1}}$

$3^{n-1} \leq 3^n$ ✓

14) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} \to \left(\left(\frac{1}{2}\right)^2\right)^n \to \left(\frac{1}{4}\right)^n$

A

$$\frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$

15) $a_n = \frac{1}{6^n}$ $\lim_{n\to\infty} \frac{1}{6^n} = 0$

D

16) $f'(x) = \sin(x^2+1) = 0$

A calc!

17) $\sum \frac{5^n}{2^n}(x+1)^n \to \left(\frac{5(x+1)}{2}\right)^n \to \left(\frac{5x+5}{2}\right)^n$

E

$\left|\frac{5x+5}{2}\right| < 1$

$-1 < \frac{5x+5}{2} < 1$

$-2 < 5x+5 < 2$

$-7 < 5x < -3$

$-\frac{7}{5} < x < -\frac{3}{5}$

18) $\sum_{k=1}^{\infty} \left(\frac{1}{5}\right)^k$ $\frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{4}$

B

19) I. $\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2}$ ✓

C

II. $\lim_{n\to\infty} \frac{e^n}{n} = \infty$

III. $\lim_{n\to\infty} \frac{e^n}{1+e^n} = 1$ ✓

20) $P\left(2 - \frac{P}{5000}\right)$

E $2P\left(1 - \frac{P}{10{,}000}\right)$

21) $\lim_{x\to\infty} f(x) = \frac{1}{4}$

E

22) $\lim_{x \to \infty} (1+5e^x)^{1/x}$

D $\ln y = \lim_{x \to \infty} \frac{1}{x} \ln(1+5e^x)$

$\ln y = \lim_{x \to \infty} \frac{\ln(1+5e^x)}{x}$ L'H

$\ln y = \lim_{x \to \infty} \frac{\frac{5e^x}{1+5e^x}}{1}$

$\ln y = 1$

$y = e$

23) $\int_0^{\pi/4} \sin x \, dx = -\cos x \Big|_0^{\pi/4} = -\frac{\sqrt{2}}{2} + 1$

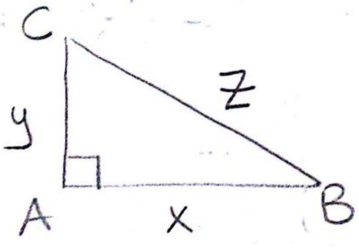
E

24) $\pi \int_0^2 [(2x)^2 - (x^2)^2] \, dx$

$\pi \int_0^2 (4x^2 - x^4) \, dx = \pi\left(\frac{4}{3}x^3 - \frac{x^5}{5}\right)\Big|_0^2$

$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \boxed{\frac{64\pi}{15}}$

25)



$\frac{dx}{dt} = 2$ in/sec

$\frac{dy}{dt} = -3$ in/sec

$\frac{dz}{dt} = ?$ when $x = 72$, $y = 96$ inches

$z = 120$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(72)(2) + 2(-3)(96) = 2(120)\left(\frac{dz}{dt}\right)$

$\frac{dz}{dt} = -\frac{6}{5}$ in/sec

26) $\lim_{n\to\infty} \left(2-\frac{1}{n}\right) = 2$ conv

27) $\lim_{n\to\infty} \frac{2n}{(n+1)(n+2)} = 0$ conv

28) $\lim_{n\to\infty} \frac{2^n+1}{2^{n+1}+1} = \frac{2^n+1}{2^n \cdot 2+1} = \frac{1}{2}$ conv

29) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$ conv

30) $\sum_{n=1}^{\infty} \frac{\pi^{3n}}{3^{\pi n}} \rightarrow \frac{(\pi^3)^n}{(3^\pi)^n} = \left(\frac{\pi^3}{3^\pi}\right)^n$

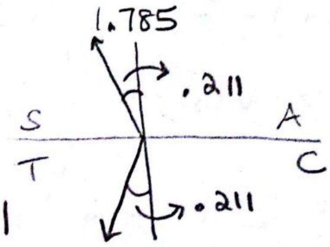
$$\frac{\frac{\pi^3}{3^\pi}}{1-\frac{\pi^3}{3^\pi}} \quad \text{or} \quad \frac{\pi^3}{3^\pi - \pi^3}$$

31) Yes, it is diff on [2, 11]

$f'(x) = \cos x \qquad \frac{f(11)-f(2)}{11-2} = \frac{-1.909}{9} = -.21214$

$\cos x = -.21214$

$x = 1.785$



$x = \frac{3\pi}{2} - 1.785 \approx 4.501$

$x = 1.785 + 2\pi \approx 8.068$

$x = 4.501 + 2\pi \approx 10.784$

$x = 1.785,\ x = 4.501,\ x = 8.068,\ x = 10.784$

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32) $\int_0^2 3x^2\sqrt{x^3+1}\,dx$ $\quad u=x^3+1$, $du=3x^2dx$

$\int u^{1/2}du = \frac{1}{2}\cdot\frac{2}{3}(x^3+1)^{3/2}\Big|_0^2$

$\frac{1}{3}(x^3+1)^{3/2}\Big|_0^2 = 9-\frac{1}{3} = \frac{26}{3}$

33) a) $v(t)=\sin(\frac{1}{2}\pi t)$ $\quad s(0)=0$

$v(1)=\sin\frac{\pi}{2}=1$

Speed at $t=1 = |v|=1$

$a(t)=v'(t)=\cos(\frac{\pi}{2}t)\cdot\frac{\pi}{2}$

$a(1)=\cos\frac{\pi}{2}\cdot\frac{\pi}{2}=0$

$s(1)=s(0)+\int_0^1\sin(\frac{\pi}{2}t)\,dt$ $\quad u=\frac{\pi}{2}t$, $du=\frac{\pi}{2}dt$, $\frac{2}{\pi}du=dt$

$s(1)=0-\frac{2}{\pi}\cos(\frac{\pi}{2}t)\Big|_0^1$ $\quad \frac{2}{\pi}\int\sin u\,du$

$s(1)=0-(\cancel{\frac{2}{\pi}\cos\frac{\pi}{2}}-\frac{2}{\pi}\cos 0)$

$s(1)=\frac{2}{\pi}$

b) $a(t)=3t$ $\quad s(0)=1,\ v(0)=0$

$v(1)=v(0)+\int_0^1 3t\,dt$ $\quad a(1)=3$

$v(1)=0+\frac{3}{2}t^2\Big|_0^1$

$v(1)=\frac{3}{2}$ $\quad$ speed at $t=1=|v|=\frac{3}{2}$

$s(1)=s(0)+\int_0^1\frac{3}{2}t^2\,dt$

$s(1)=1+\frac{1}{2}t^3\Big|_0^1=1+\frac{1}{2}=\frac{3}{2}$

34) a) $\frac{1}{(e+1)-2} \int_2^{e+1} \ln(x-1)\,dx \approx .3387$

b) $\int_2^{e+1} \ln(x-1)\,dx = 1$ $\qquad \int_2^{K} \ln(x-1)\,dx = \frac{1}{2}$

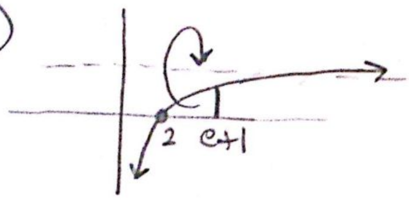
$\int \ln x = x\ln x - x + C$

$(x-1)\ln(x-1) - (x-1)\Big|_2^K = \frac{1}{2}$

$(K-1)\ln(K-1) - (K-1) - (1\ln 1 - (1))$

$(K-1)\ln(K-1) - K + 1 - 0 + 1$

$(K-1)\ln(K-1) - K + 2 = \frac{1}{2}$ $\qquad K \approx 3.156$

c) 

$\pi \int_2^{e+1} \left[1^2 - (1 - \ln(x-1))^2\right] dx$

$1.282\pi \approx 4.027$

35) a) $y = \frac{e^{2-2x^2}}{x}$

$y' = \frac{x(e^{2-2x^2})(-4x) - (e^{2-2x^2})}{x^2} = \frac{(-4x^2-1)(e^{2-2x^2})}{x^2}$

b) $f'(x)$ $\quad - \quad | \quad -$ $\qquad (-\infty, 0) \cup (0, \infty)$

x $\qquad 0$

c) No pts of inflection

VA at $x=0$

d) CC↑ on $(0, \infty)$

$$y'' = \frac{x^2\left[(e^{2-2x^2})(-8x) + (-4x^2-1)(-4x(e^{2-2x^2}))\right] - \left[(e^{2-2x^2})(-4x^2-1)\cdot 2x\right]}{x^4}$$

$f''(x)$: $-$ | $+$

x: 0

$$f''(1) = 1\left[(e^0)(-8) + (-4-1)(-4\cdot 1)\right] - \left[1\cdot(-4-1)\cdot 2\right]$$

$$-8 + (-5)(-4) - (-10)$$

$$-8 + 20 + 10$$

36)

a) $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \quad \frac{1}{1-\frac{1}{2}} = 2$

b) $\left|\frac{t}{t+1}\right| < 1$

$-1 < \frac{t}{t+1} < 1$

$-t-1 < t < t+1$

$-2t-1 < 0 < 1$

$-2t < 1$

$t > -\frac{1}{2}$

$\left(-\frac{1}{2}, \infty\right)$

c) $\text{sum} = \frac{1}{1-\frac{t}{t+1}} > 10$

$1 > 10\left(1 - \frac{t}{t+1}\right)$

$1 > 10 - \frac{10t}{t+1}$

$t+1 > 10(t+1) - 10t$

$t+1 > \cancel{10t} + 10 - \cancel{10t}$

$t+1 > 10$

$t > 9$

37) $\sum_{n=1}^{\infty} \frac{4n}{n^2+1}$ Compare to $\frac{4}{n}$ div
a)

LCT: $\lim_{n\to\infty} \frac{4n}{n^2+1} \cdot \frac{n}{4} = 1 \therefore$ div

b) $\sum_{n=1}^{\infty} \left(\frac{4n}{n^2+1}\right)\left(\frac{1}{2n}\right) \to \sum_{n=1}^{\infty} \frac{4n}{2n^3+2n}$ Compare to $\frac{4}{n^2}$ conv

c) LCT: $\lim_{n\to\infty} \frac{4n}{2n^3+2n} \cdot \frac{n^2}{4} = 1 \therefore$ conv

38) a) $R'(2) \approx \frac{R(3)-R(1)}{3-1} = -120$ liters/hr^2

b) $\int_0^8 R(t)dt \approx 1(1340) + 2(1190) + 3(950) + 2(740)$
$= 8050$ liters

c) Total $\approx 50{,}000 + \int_0^8 W(t)dt - 8050$

$50000 + 7836.195325 - 8050$

≈ 49786 liters

d) $W(0) - R(0) > 0$
$W(8) - R(8) < 0$
$W(t) - R(t)$ is continuous
$\therefore$ IVT guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$ or $W(t) = R(t)$
For this value of t, the rate at which water is being pumped in is same as rate it is removed

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