FRQ 1: Inactive

AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 5

Let f and g be the functions defined by $f(x) = \frac{1}{x}$ and $g(x) = \frac{4x}{1+4x^2}$, for all $x > 0$.

(a) Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0, \infty)$ if the minimum exists. Justify your answers.

(b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x = 1$, below the graph of f, and above the graph of g.

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FRQ 2: Active

AP® CALCULUS BC
2006 SCORING GUIDELINES

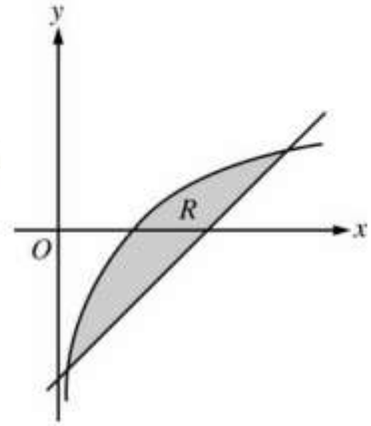
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



FRQ 2: Active

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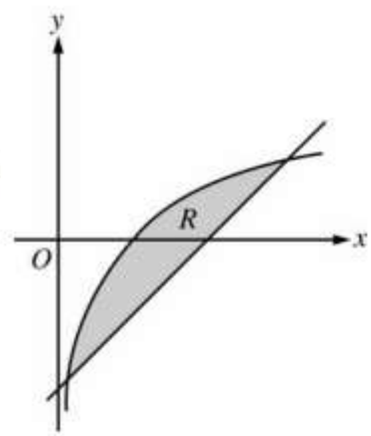

FRQ 3: Inactive

AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x)+4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

FRQ 3: Inactive

AP® CALCULUS AB 2002 SCORING GUIDELINES

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x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
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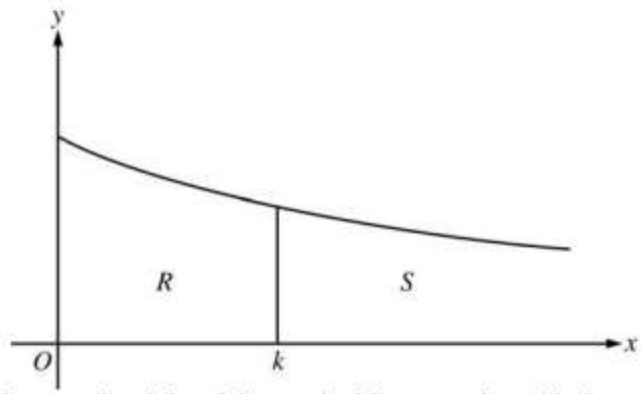
The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

FRQ 4: Inactive

AP® CALCULUS BC
2005 SCORING GUIDELINES (Form B)

Question 6



Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f, the x- and y-axes, and the vertical line $x = k$, where $k \geq 0$.

(a) Find the area of R in terms of k.

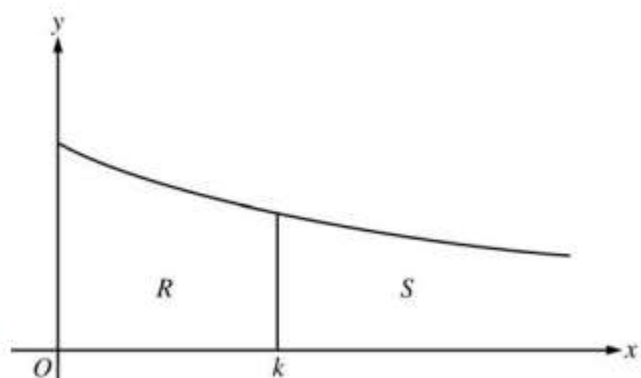
(b) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f, as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (b).

FRQ 4: Inactive

AP® CALCULUS BC
2005 SCORING GUIDELINES (Form B)

Question 6



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FRQ 1 Rubric

(a) $g'(x) = \dfrac{4(1+4x^2) - 4x(8x)}{(1+4x^2)^2} = \dfrac{4(1-4x^2)}{(1+4x^2)^2}$

For $x > 0$, $g'(x) = 0$ for $x = \frac{1}{2}$.

$g'(x) > 0$ for $0 < x < \frac{1}{2}$

$g'(x) < 0$ for $x > \frac{1}{2}$

$g\left(\frac{1}{2}\right) = 1$

Therefore g has a maximum value of 1 at $x = \frac{1}{2}$, and g has no minimum value on the open interval $(0, \infty)$.

$$5 : \begin{cases} 2 : g'(x) \\ 1 : \text{critical point} \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$$

(b) $\int_1^{\infty} (f(x) - g(x))\, dx = \lim_{b \to \infty} \int_1^{b} (f(x) - g(x))\, dx$

$= \lim_{b \to \infty} \left(\ln(x) - \frac{1}{2}\ln(1 + 4x^2) \right) \Big|_{x=1}^{x=b}$

$= \lim_{b \to \infty} \left(\ln(b) - \frac{1}{2}\ln(1 + 4b^2) + \frac{1}{2}\ln(5) \right)$

$= \lim_{b \to \infty} \ln\left(\dfrac{b\sqrt{5}}{\sqrt{1 + 4b^2}} \right)$

$= \lim_{b \to \infty} \ln\left(\dfrac{\sqrt{5b^2}}{\sqrt{1 + 4b^2}} \right)$

$= \dfrac{1}{2} \lim_{b \to \infty} \ln\left(\dfrac{5b^2}{1 + 4b^2} \right)$

$= \dfrac{1}{2}\ln\dfrac{5}{4}$

$$4 : \begin{cases} 1 : \text{integral} \\ 2 : \text{antidifferentiation} \\ 1 : \text{answer} \end{cases}$$

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FRQ 2 Rubric

$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

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FRQ 3 Rubric

(a) $\int_0^{1.5} (3f'(x)+4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$ $= 3f(x) + 4x\Big\vert_0^{1.5} = 3(-1-(-7)) + 4(1.5) = 24$	2 { 1 : antiderivative; 1 : answer
(b) $y = 5(x-1) - 4$ $f(1.2) \approx 5(0.2) - 4 = -3$ The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.	3 { 1 : tangent line; 1 : computes y on tangent line at $x = 1.2$; 1 : answer with reason
(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that $f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3-0}{0.5} = 6 = r$	2 { 1 : reference to MVT for f' (or differentiability of f'); 1 : value of r for interval $0 \le x \le 0.5$
(d) $\lim_{x\to 0^-} g'(x) = \lim_{x\to 0^-} (4x-1) = -1$ $\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} (4x+1) = +1$ Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$. OR $g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.	2 { 1 : answers "no" with reference to g' or g''; 1 : correct reason

FRQ 3 Rubric

(a) $\int_0^{1.5} (3f'(x) + 4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1-(-7)) + 4(1.5) = 24$

2 { 1 : antiderivative; 1 : answer }

(b) $y = 5(x-1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

3 { 1 : tangent line; 1 : computes y on tangent line at $x = 1.2$; 1 : answer with reason }

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$$f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3 - 0}{0.5} = 6 = r$$

2 { 1 : reference to MVT for f' (or differentiability of f'); 1 : value of r for interval $0 \le x \le 0.5$ }

(d) $\lim_{x\to 0^-} g'(x) = \lim_{x\to 0^-} (4x - 1) = -1$

$\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

2 { 1 : answers "no" with reference to g' or g''; 1 : correct reason }

FRQ 4 Rubric

(a) Area of $R = \int_0^k \frac{1}{x+2}\,dx = \ln(k+2) - \ln(2)$

2 : { 1 : integral; 1 : antidifferentiation and evaluation }

(b) $V_R = \pi \int_0^k \frac{1}{(x+2)^2}\,dx$

$= -\frac{\pi}{x+2}\Big|_0^k = \frac{\pi}{2} - \frac{\pi}{k+2}$

3 : { 1 : limits; 1 : integrand; 1 : antidifferentiation and evaluation }

(c) $V_S = \pi \int_k^\infty \frac{1}{(x+2)^2}\,dx$

$= \lim_{n\to\infty} -\frac{\pi}{x+2}\Big|_k^n = \frac{\pi}{k+2}$

$V_S = V_R$

$\frac{\pi}{k+2} = \frac{\pi}{2} - \frac{\pi}{k+2}$

$\frac{2}{k+2} = \frac{1}{2}$

$k = 2$

4 : { 1 : improper integral; 1 : antidifferentiation and evaluation; 1 : equation; 1 : answer }

FRQ 4 Rubric

(a) Area of $R = \int_0^k \frac{1}{x+2}\,dx = \ln(k+2) - \ln(2)$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{antidifferentiation and evaluation} \end{cases}$

(b) $V_R = \pi \int_0^k \frac{1}{(x+2)^2}\,dx$

$= -\frac{\pi}{x+2}\Big|_0^k = \frac{\pi}{2} - \frac{\pi}{k+2}$

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{antidifferentiation and evaluation} \end{cases}$

(c) $V_S = \pi \int_k^{\infty} \frac{1}{(x+2)^2}\,dx$

$= \lim_{n \to \infty} -\frac{\pi}{x+2}\Big|_k^n = \frac{\pi}{k+2}$

$V_S = V_R$

$\frac{\pi}{k+2} = \frac{\pi}{2} - \frac{\pi}{k+2}$

$\frac{2}{k+2} = \frac{1}{2}$

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$4: \begin{cases} 1: \text{improper integral} \\ 1: \text{antidifferentiation and evaluation} \\ 1: \text{equation} \\ 1: \text{answer} \end{cases}$