1969 AB2/BC2

A particle moves along the x-axis in such a way that its position at time t is given by $x = 3t^4 - 16t^3 + 24t^2$ for $-5 \le t \le 5$.

(a) Determine the velocity and acceleration of the particle at time t.

(b) At what values of t is the particle at rest?

(c) At what values of t does the particle change direction?

(d) What is the velocity when the acceleration is first zero?

1975 AB5

The line $x = c$ where $c > 0$ intersects the cubic $y = 2x^3 + 3x^2 - 9$ at point P and the parabola $y = 4x^2 + 4x + 5$ at point Q.

(a) If a line tangent to the cubic at point P is parallel to the line tangent to the parabola at point Q, find the value of c where $c > 0$.

(b) Write the equations of the two tangent lines described in (a).

1976 AB1

Let f be the real-valued function defined by $f(x) = \sqrt{1 + 6x}$.

(a) Give the domain and range of f.

(b) Determine the slope of the line tangent to the graph of f at $x = 4$.

(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.

(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

1977 AB4/BC2

Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f' and g' at $x = 1$ and $x = 2$ be given by the table below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	5	4
2	2	π	6	7

Determine the value of each of the following.

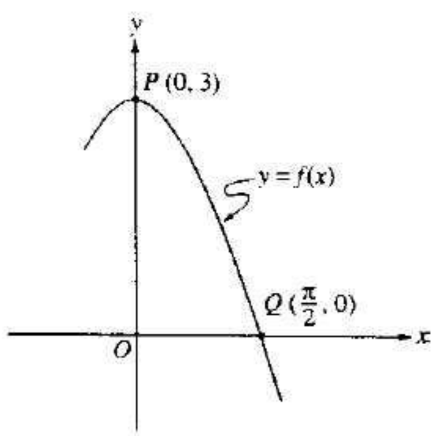
(a) The derivative of $f + g$ at $x = 2$

(b) The derivative of fg at $x = 2$

(c) The derivative of $\dfrac{f}{g}$ at $x = 2$

(d) $h'(1)$ where $h(x) = f(g(x))$

1997 AB2



Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at point P and the x-axis at point Q.

(a) Write an equation for the line passing through the points P and Q.

(b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your conclusion.

(c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.

1969 AB2/BC2
Solution

(a) $v = \dfrac{dx}{dt} = 12t^3 - 48t^2 + 48t = 12t(t^2 - 4t + 4) = 12t(t-2)^2$

$a = \dfrac{dv}{dt} = 36t^2 - 96t + 48 = 12(3t^2 - 8t + 4) = 12(3t-2)(t-2)$

(b) The particle is at rest when $v = 0$. This occurs when $t = 0$ and $t = 2$.

(c) The particle changes direction at $t = 0$ only.

(d) $a = 0$ when $t = \dfrac{2}{3}$ and $t = 2$. The acceleration is first zero at $t = \dfrac{2}{3}$.

$v\left(\dfrac{2}{3}\right) = 12\left(\dfrac{2}{3}\right)\left(-\dfrac{4}{3}\right)^2 = \dfrac{128}{9}$

(a) $y' = 6x^2 + 6x$ for the cubic
The slope of the tangent line at point *P*, where $x = c$, is $6c^2 + 6c$.

$y' = 8x + 4$ for the parabola
The slope of the tangent line at point *Q*, where $x = c$, is $8c + 4$.

Since the two lines are parallel,
$6c^2 + 6c = 8c + 4$
$6c^2 - 2c - 4 = 0$
$2(3c + 2)(c - 1) = 0$

Since $c > 0$, the solution is $c = 1$.

(b) tangent to cubic:

The slope is $m = 12$ and the line contains $(1, -4)$.
Therefore the equation is $y + 4 = 12(x-1)$, or $y = 12x - 16$.

tangent to parabola:

The slope is $m = 12$ and the line contains $(1, 13)$.
Therefore the equation is $y - 13 = 12(x-1)$, or $y = 12x + 1$.

1976 AB1
Solution

(a) The domain of f is $x \geq -\frac{1}{6}$.

The range of f is $y \geq 0$.

(b) $f'(x) = \frac{3}{\sqrt{1+6x}}$

The slope of the tangent line at $x = 4$ is $f'(4) = \frac{3}{5}$.

(c) $f(4) = 5$

The tangent line is $y - 5 = \frac{3}{5}(x - 4)$

Therefore the y-intercept is at $y = \frac{13}{5}$.

(d) The tangent line parallel to $y = x + 12$ has slope 1.

$$f'(x) = \frac{3}{\sqrt{1+6x}} = 1$$

$$9 = 1 + 6x$$

$$x = \frac{4}{3}$$

$$y = \sqrt{1 + 6\left(\frac{4}{3}\right)} = 3$$

The coordinates of the point are $\left(\frac{4}{3}, 3\right)$.

1977 AB4/BC2
Solution

(a) $(f+g)'(2) = f'(2) + g'(2) = 6 + 7 = 13$

(b) $(fg)'(2) = f(2)g'(2) + f'(2)g(2) = 2 \cdot 7 + 6 \cdot \pi = 14 + 6\pi$

(c) $\left(\dfrac{f}{g}\right)'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \dfrac{\pi \cdot 6 - 2 \cdot 7}{\pi^2} = \dfrac{6\pi - 14}{\pi^2}$

(d) $(f \circ g)'(1) = f'(g(1))g'(1) = f'(2) \cdot 4 = 6 \cdot 4 = 24$

1997 AB2
Solution

(a) $\text{slope} = \dfrac{3-0}{0-\pi/2} = -\dfrac{6}{\pi}$

$$y-3 = -\frac{6}{\pi}(x-0)$$

(b) $f'(x) = -3\sin x$

$$f'(\pi/2) = -3\sin(\pi/2) = -3$$

$$y-0 = -3(x-\pi/2)$$

(c) $f'(x) = -3\sin x = -\dfrac{6}{\pi}$

$$\sin x = \frac{2}{\pi}$$

$$x = 0.690$$