KEY

LAYING the FOUNDATION
Advanced Programs Division

Practice Problems

Limit as x approaches infinity

1. $\lim_{x\to\infty}\left(\frac{3x-7}{5x^4-8x+12}\right)= 0$

2. $\lim_{x\to\infty}\left(\frac{3x^4-2}{5x^4-2x+1}\right)= \frac{3}{5}$

3. $\lim_{x\to\infty}\left(\frac{x^6-2}{10x^4-9x+8}\right)= \infty$

4. $\lim_{x\to\infty}\left(\frac{7x^4-2}{5-2x^3-14x^4}\right)= \frac{-7}{14} = \frac{-1}{2}$

5. $\lim_{x\to\infty}\left(\frac{\sin x}{e^x}\right)= 0$

6. $\lim_{x\to-\infty}\left(\frac{\sqrt{x^2-9}}{2x-3}\right)= \frac{-1}{2}$

7. $\lim_{x\to\infty}\left(\frac{\sqrt{x^2-9}}{2x-3}\right)= \frac{1}{2}$

Limit as x approaches a number

8. $\lim_{x\to 2}\left(x^3-x+1\right)$ $2^3-2+1=7$

9. $\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right)= \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = x+2$
$2+2=4$

10. $\lim_{x\to 2^-}\left(\frac{3}{x-2}\right)= -\infty$

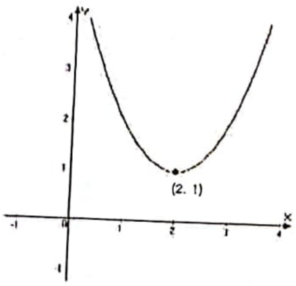
11. $\lim_{x\to 2^+}\left(\frac{3}{x-2}\right)= \infty$

12. $\lim_{x\to 2}\left(\frac{3}{x-2}\right)=$ DNE

13. $\lim_{x\to 2^+}\left(\frac{3}{2-x}\right)= -\infty$

14. $\lim_{x\to\frac{\pi}{4}}\left(\frac{\sin x}{x}\right)= \frac{\sin\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\pi}{4}}$
$\frac{\sqrt{2}}{2}\cdot\frac{4}{\pi} = \frac{2\sqrt{2}}{\pi}$

15. $\lim_{x\to\frac{\pi}{4}}\left(\frac{\tan x}{x}\right)= \frac{\tan\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{1}{\frac{\pi}{4}}$
$= \frac{4}{\pi}$



$(f(2))^3 - 3f(2) + 7$
$1^3 - 3(1) + 7 = 1 - 3 + 7$
5

4. The graph of $y = f(x)$ is shown above. $\lim_{x \to 2}\left(\left(f(x)^3\right) - 3f(x) + 7\right) =$

(A) 1 (B) 5 (C) 7 (D) 9 (E) Does not exist

5. If $f(x) = \begin{cases} \dfrac{x^2 - 3x - 4}{x+1}, & x \neq -1 \\ 2, & x = -1 \end{cases}$, what is $\lim_{x \to -1} f(x)$?

$\dfrac{(x-4)\cancel{(x+1)}}{\cancel{(x+1)}} = x - 4$
$-1 - 4 = -5$

(A) -5 (B) 0 (C) 2 (D) 3 (E) Does not exist

6. $\lim_{x \to \infty}\left(\dfrac{2x^6 - 5x^3 + 10}{20 - 4x^2 - x^6}\right) =$ -2

(A) -2 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 2 (E) Does not exist

7. $\lim_{x \to \infty}\left(\dfrac{2x^5 - 5x^3 + 10}{20 - 4x^2 - x^6}\right) =$

(A) -2 (B) $-\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 2

8. $\lim_{x\to\infty}\left(1+e^{\frac{1}{2}+\frac{1}{x}}\right)=$ $1+e^{\frac{1}{2}+0}$

(A) $-\infty$ (B) 0   (C) $e^{\frac{1}{2}}$

(D) $1+e^{\frac{1}{2}}$ (E) ∞

9. $\lim_{x\to 3^+}\frac{5}{3-x}=$

(A) $-\infty$ (B) -5 (C) 0

(D) $\frac{5}{3}$ (E) ∞

10. If $\lim_{x\to\infty}\left(\frac{5n^3}{20-3n-kn^3}\right)=\frac{1}{2}$, then $k=$ $\frac{-5}{K}=\frac{1}{2}$ $-5=\frac{1}{2}K$ $K=-10$

(A) -10 (B) -4 (C) $\frac{1}{4}$ (D) 4   (E) 10

11. Which of the following is/are true about the function g if $g(x)=\frac{(x-2)^2}{x^2+x-6}$? $\frac{(x-2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ $\frac{x-2}{x+3}$

~~I.~~ g is continuous at $x=2$ hole
II. The graph of g has a vertical asymptote at $x=-3$ T
~~III.~~ The graph of g has a horizontal asymptote at $y=0$

(A) I only   (B) II only   (C) III only   (D) I and II only   (E) II and III only

12. $f(x)=\begin{cases}\sin x, & x<\frac{\pi}{4} \quad \frac{\sqrt{2}}{2}\\ \cos x, & x>\frac{\pi}{4} \quad \frac{\sqrt{2}}{2}\\ \tan x, & x=\frac{\pi}{4} \quad 1\end{cases}$

What is $\lim\limits_{x\to\frac{\pi}{4}} f(x)$?

(A) $-\infty$ (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) ∞

13. $\lim\limits_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{x-a}\right)=$ $\frac{(\sqrt{x}-\sqrt{a})}{x-a}\cdot\frac{(\sqrt{x}+\sqrt{a})}{\sqrt{x}+\sqrt{a}}=\frac{\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})}=\frac{1}{\sqrt{x}+\sqrt{a}}$

(A) $\frac{1}{2\sqrt{a}}$ (B) $\frac{1}{\sqrt{a}}$ (C) $\sqrt{a}$ (D) $2\sqrt{a}$ (E) Does not exist

$\frac{1}{\sqrt{a}+\sqrt{a}}$

$\frac{1}{2\sqrt{a}}$

14. $\lim\limits_{x\to 0^+}\frac{\ln 2x}{2x}=$ $\frac{-\infty}{\text{small}}$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

$\frac{x^2+3x}{3x+2}$

Free Response 2 (No calculator)

Given the function $f(x)=\frac{x^3+2x^2-3x}{3x^2+3x-6}$. $\frac{x(x^2+2x-3)}{3(x^2+x-2)}=\frac{x(x+3)\cancel{(x-1)}}{3(x+2)\cancel{(x-1)}}$

(a) What are the zeros of $f(x)$? $x=0,\ x=-3$

(b) What are the vertical asymptotes of $f(x)$? $x=-2$

(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

$$\begin{array}{r} \frac{1}{3}x \\ 3x+2\,\overline{)\,x^2+3x} \\ -(x^2+\frac{2}{3}x) \\ \hline \frac{7}{3}x \end{array}$$

$y=\frac{1}{3}x$

(d) What is $\lim\limits_{x\to\infty} f(x)$? What is $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}$? 1

∞

Answers to Unit 1 – Limit and Continuity Review

Practice Problems:

1. 0
2. $\frac{3}{5}$
3. ∞
4. $-\frac{1}{2}$
5. 0
6. $-\frac{1}{2}$
7. $\frac{1}{2}$
8. 7
9. 4
10. $-\infty$
11. ∞
12. does not exist
13. $-\infty$
14. $\frac{2\sqrt{2}}{\pi}$
15. $\frac{4}{\pi}$

5. A
6. A
7. C
8. D
9. A
10. A
11. B
12. D
13. A
14. A

Free Response 2 (No calculator)

Given the function $f(x)=\dfrac{x^3+2x^2-3x}{3x^2+3x-6}$.

(a) What are the zeros of $f(x)$?

(b) What are the vertical asymptotes of $f(x)$?

(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

(d) What is $\lim\limits_{x\to\infty} f(x)$? What is $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}$?

(a) The zeros of the function, $f(x)$, occur at $x=-3,\ 0,\ 1$	3 pts, 1 for each zero
(b) There is a vertical asymptote at $x=-2$	1 pt for the vertical asymptote
(c) $g(x)=\dfrac{1}{3}x$	2 pts for $g(x)$
(d) $\lim\limits_{x\to\infty} f(x)=\infty$	1 pt $\lim\limits_{x\to\infty} f(x)$
$\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}=1$	2 pts for $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}$

$$\lim_{x\to\infty}\frac{x^2+3x}{3x+2}\cdot\frac{3}{x}=1$$