AP CALCULUS AB Problem Set Unit 1 Name: ____________________________

1. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

2. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$ (E) $y = \left(x^2+1\right)^2 - 1$

3. Let $f(x) = \cos(\arctan x)$. What is the range of f?

(A) $\left\{x \middle| -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$ (B) $\{x | 0 < x \le 1\}$ (C) $\{x | 0 \le x \le 1\}$

(D) $\{x | -1 < x < 1\}$ (E) $\{x | -1 \le x \le 1\}$

4. $\lim\limits_{n \to \infty} \frac{4n^2}{n^2 + 10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

5. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

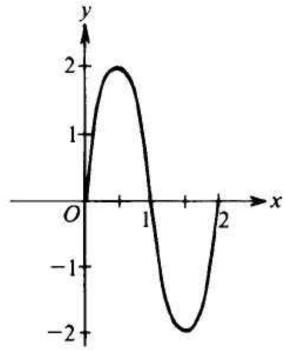
(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

6. Which of the following functions are continuous for all real numbers x?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

7.



The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

8. If $f(x) = 2x^2 + 1$, then $\lim\limits_{x \to 0} \frac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

9. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

10. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

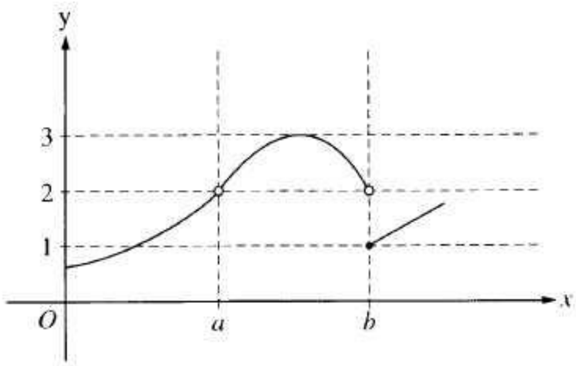
(D) 225 (E) 5

11. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only

(D) II and III only (E) I, II, and III

12. 

The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 2$

(C) $\lim_{x\to b} f(x) = 2$

(D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

13. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

14. If $a \neq 0$, then $\lim_{x\to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

Let f be the function that is given by $f(x) = \frac{ax+b}{x^2-c}$ and that has the following properties.

i) The graph of f is symmetric with respect to the y-axis

ii) $\lim_{x \to 2^+} f(x) = +\infty$

iii) $f(1) = -3$

a) Determine the values of a, b, and c.

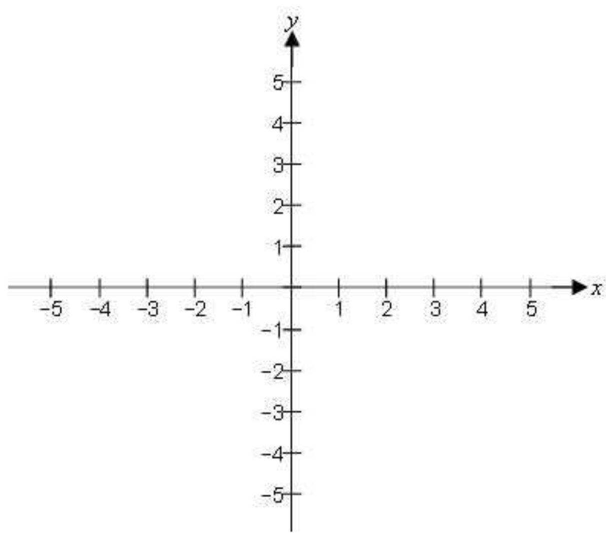
b) Write an equation for each vertical and each horizontal asymptote of the graph of f. Justify your answer.

c) Sketch the graph of f in the xy-plane provided below.

a)

b)

c)



Answers:

1. A	**2. B**		**3. B**	**4. D**	**5. E**	**6. D**	**7. D**
	8. C	**9. C**	**10. E**	**11. C**	**12. B**	**13. E**	**14. B**

a) Graph symmetric to y-axis $\Rightarrow f$ is even

$f(-x) = f(x)$ therefore $a = 0$

$\lim_{x \to 2^+} f(x) = +\infty$ therefore $c = 4$

$$f(x) = \frac{b}{x^2 - 4}$$

$f(1) = -3$, therefore b=9

b)

$$f(x) = \frac{9}{x^2 - 4}$$

Vertical: $x = 2$, $x = -2$

Horizontal: $y = 0$

Justify: the function is undefined at $x = \pm 2$. The limit as x approaches 2 from the right is infinity and the left is negative infinity. The limit as x approaches -2 from the left is infinity and from the right is negative infinity. This means vertical asymptotes occur here. The limit as x approaches positive and negative infinity is 0, therefore, the horizontal asymptote occurs at y=0.

c)