Unit 1 MC Review:

1) D

$$\int_0^3 (x+1)^{1/2}dx$$

$u=x+1$
$du=dx$
$u(0)=1$
$u(3)=4$

$$\int_1^4 u^{1/2}du$$

$$\frac{2}{3}u^{3/2}\Big|_1^4$$

$$\frac{2}{3}(4^{3/2}-1^{3/2})$$

$$\frac{2}{3}(8-1)$$

$$\frac{14}{3}$$

2) E

$$\int x\sqrt{4-x^2}\,dx$$

$u=4-x^2$
$du=-2x\,dx$
$-\frac{1}{2}du=x\,dx$

$$-\frac{1}{2}\int u^{1/2}du$$

$$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

$$-\frac{1}{3}(4-x^2)^{3/2}+C$$

3) B

$$\int_0^1 (x+1)\cdot e^{x^2+2x}dx$$

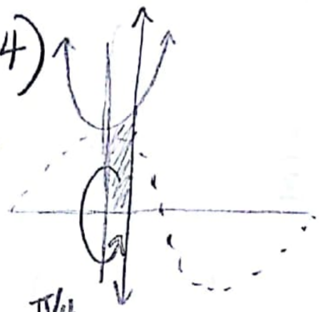
$u=(x^2+2x)$
$du=(2x+2)dx$
$du=2(x+1)dx$
$\frac{1}{2}du=(x+1)dx$
$u(0)=0$
$u(1)=3$

$$\frac{1}{2}\int_0^3 e^u du$$

$$\frac{1}{2}e^u\Big|_0^3$$

$$\frac{1}{2}(e^3-1)$$

4) C



$$\pi\int_0^{\pi/4}\sec^2x\,dx$$

$$\pi\cdot(\tan x)\Big|_0^{\pi/4}$$

$$\pi\left(\tan\frac{\pi}{4}-\tan 0\right)$$

$$\pi(1-0)=\pi$$

5) E

$$\int_0^1 \frac{x+1}{x^2+2x-3}dx$$

$(x+3)(x-1)$

$$\frac{x+1}{(x+3)(x-1)}=\frac{A}{(x+3)}+\frac{B}{(x-1)}$$

$$x+1=A(x-1)+B(x+3)$$

$x=1:\ 2=B\cdot 4 \quad B=\frac{1}{2}$

$x=-3:\ -2=A\cdot(-4)\ A=\frac{1}{2}$

$$\lim_{b\to 1^-}\int_0^b\left[\frac{1}{2}\cdot\frac{1}{(x+3)}+\frac{1}{2}\cdot\frac{1}{(x-1)}\right]dx$$

$$\lim_{b\to 1^-}\frac{1}{2}\int_0^b\left(\frac{1}{x+3}+\frac{1}{x-1}\right)dx$$

$$\lim_{b\to 1^-}\frac{1}{2}\cdot\left(\ln|x+3|+\ln|x-1|\Big|_0^b\right)$$

$$\lim_{b\to 1^-}\frac{1}{2}\left(\ln|b+3|+\ln|b-1|-\ln 3\right)$$

$$\frac{1}{2}(\ln 4+-\infty-\ln 3)\ \therefore \text{div}$$

6) $\int_{-1}^{1} e^{-x^2}dx = K$ e^{-x^2} is even

D

$\int_{-1}^{0} e^{-x^2}dx = \frac{K}{2}$

7) $\int_{-2}^{2}(x^7+K)dx = 16$

D

$\frac{x^8}{8} + Kx \Big|_{-2}^{2} = 16$

$(32+2K)-(32-2K)=16$

$4K=16$

$K=4$

8) $x(t) = -5t^2$

C

$\frac{x(3)-x(0)}{3-0} = \frac{-45-0}{3}$

-15

9) $\lim_{x\to 0} x \cdot \csc x$

D

$\lim_{x\to 0} \frac{x}{\sin x} \to \frac{0}{0}$ L'H

$\lim_{x\to 0} \frac{1}{\cos x} = 1$

10) $\frac{d}{dx}\int_{2}^{x}\sqrt{1+t^2}\,dt$

C

$\sqrt{1+x^2}$

11) $\int \frac{1}{(x-1)(x+2)}dx$

A

$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$1 = A(x+2) + B(x-1)$

$x=-2$: $1=-3B$ $B=-\frac{1}{3}$

$x=1$: $1=3A$ $A=\frac{1}{3}$

$\frac{1}{3}\int\frac{1}{x-1}dx - \frac{1}{3}\int\frac{1}{x+2}dx$

$\frac{1}{3}(\ln|x-1| - \ln|x+2|) + C$

$\frac{1}{3} \cdot \ln\left|\frac{x-1}{x+2}\right| + C$

12) $v(t) = 2t-4$

C

$x(0)=4$

$x(t) = \int(2t-4)dt$

$x(t) = t^2-4t+C$

$4 = 0-0+C$

$C=4$

$x(t) = t^2-4t+4$

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13) $g(x) = e^{f(x)}$

D $g''(x) = h(x)e^{f(x)}$

$g'(x) = e^{f(x)} \cdot f'(x)$

$g''(x) = e^{f(x)} \cdot f''(x) + f'(x) \cdot e^{f(x)} \cdot f'(x)$

$g''(x) = e^{f(x)}\left(f''(x) + (f'(x))^2\right)$

$h(x) = f''(x) + (f'(x))^2$

14) $\lim\limits_{h \to 0} \dfrac{\int_1^{1+h} \sqrt{x^5+8}\, dx}{h} \to \dfrac{0}{0}$ ∴ L'H

C

$\lim\limits_{h \to 0} \dfrac{\sqrt{(1+h)^5+8}}{1} = 3$

15) C → concavity changes

16) $\frac{dA}{dt} = 96\pi \ m^2/s$

A When $A = 64\pi$

$\frac{dr}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$96\pi = 2\pi(8)\left(\frac{dr}{dt}\right)$

$6 = \frac{dr}{dt}$

6 m/s

$A = \pi r^2$

$64\pi = \pi r^2$

$r = 8$

17) $\lim\limits_{x \to \infty} (1 + 5e^x)^{1/x}$ ∞^0

C

$\ln y = \lim\limits_{x \to \infty} \frac{1}{x} \ln(1 + 5e^x)$ $0 \cdot \infty$

$\ln y = \lim\limits_{x \to \infty} \dfrac{\ln(1+5e^x)}{x}$ $\frac{\infty}{\infty}$ ∴ L'H

$\ln y = \lim\limits_{x \to \infty} \dfrac{\frac{1}{1+5e^x} \cdot 5e^x}{1}$ $\frac{\infty}{\infty}$ ∴ L'H

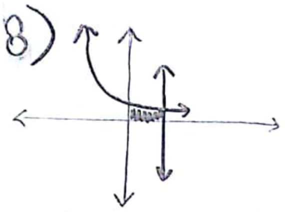
$\ln y = \lim\limits_{x \to \infty} \dfrac{5e^x}{5e^x}$

$\ln y = 1$

$y = e$

18)

A



$A_{square} = (e^{-x})^2 = e^{-2x}$

$V = \int_0^3 e^{-2x}\,dx$

$-\frac{1}{2}e^{-2x}\Big|_0^3$

$-\frac{1}{2}(e^{-6} - 1)$

$\frac{-e^{-6}+1}{2}$

19) E

20) $\frac{dy}{dt} = -2y$

C

$y=1,\ t=0$

$y=\frac{1}{2},\ t=?$

$\int \frac{1}{y}\,dy = \int (-2)\,dt$

$\ln|y| = -2t + c$

$|y| = e^{-2t+c}$

$y = \pm e^{-2t+c}$

$y = Ce^{-2t}$

$1 = Ce^0$

$C = 1$

$y = e^{-2t}$

$\frac{1}{2} = e^{-2t}$

$\ln\frac{1}{2} = -2t$

$t = \frac{\ln\frac{1}{2}}{-2} = \frac{\ln 2}{2}$

21) $f''(x)$ changes neg to pos

D at $x = -1.894$

22) $\int_1^3 f(x)\,dx = 2.3$

D

$F(3) - F(1) = 2.3$

$\int_0^1 f(x)\,dx = 2$

$F(1) - F(0) = 2$

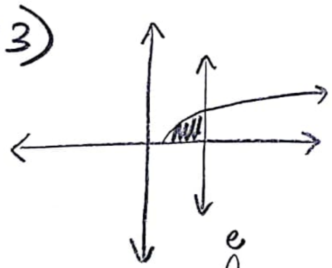
$F(1) = F(0) + 2$

$F(3) - (F(0)+2) = 2.3$

$F(3) - F(0) = 4.3$

23)

C



$A_{square} = (\sqrt{\ln x})^2 = \ln x$

$\int_1^e \ln x\,dx$

$x\ln x - x\Big|_1^e$

$(e\cdot\ln e - e) - (1\cdot\ln 1 - 1)$

$-(-1) = 1$

24) $f'(x)$ goes pos to neg

C $x = .910$

25) $v(t) = \int (t + \sin t)\,dt$

B

$v(t) = \frac{t^2}{2} - \cos t + c$

$-2 = 0 - 1 + c$

$-1 = c$

$v(t) = 0 = \frac{t^2}{2} - \cos t - 1$

$t = 1.478$