AP Calculus AB Unit 1 Free Response Practice KEY

1971 AB1
Solution

$H(x) = f(g(x)) = \ln(x^2 - 4)$

(a) The domain of H is the set of x for which $x^2 - 4 > 0$, that is, $x > 2$ or $x < -2$. There are other equivalent ways to write this set.

(b) The range of H is the set of all real numbers.

$K(x) = g(f(x)) = (\ln x)^2 - 4$

(c) The domain of K is the same as the domain of the natural logarithm, that is, $x > 0$.

(d) The range of K is the set of all real numbers ≥ -4.

(e) $H'(x) = f'(g(x))g'(x) = \dfrac{1}{g(x)} \cdot 2x = \dfrac{2x}{x^2 - 4}$

$H'(7) = \dfrac{14}{45}$

1973 AB1
Solution

(a)
$$x^3 - 6x^2 + 9x = 4$$
$$x^3 - 6x^2 + 9x - 4 = 0$$
$$(x-1)(x^2 - 5x + 4) = 0$$
$$(x-1)(x-1)(x-4) = 0$$

1	1	−6	9	−4
		1	−5	4
	1	−5	4	0

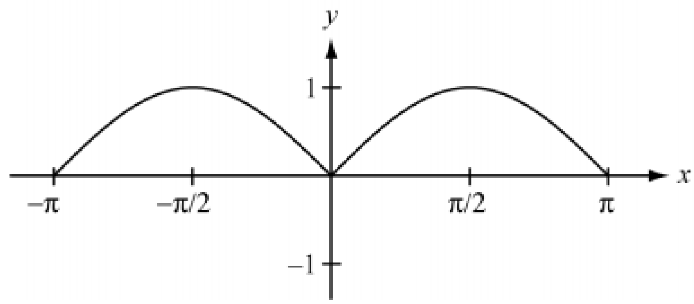
The roots are $x = 1$ and $x = 4$. The coordinates of the common points are therefore $(1, 4)$ and $(4, 4)$.

(b)
$$x^3 - 6x^2 + 9x = 0$$
$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)^2 = 0$$

The zeros of f are $x = 0$ and $x = 3$.

1974 AB1/BC1
Solution

(a)



(b) $H(x) = (|\sin x|)^2 = \sin^2 x$

(c) The domain of H is $-\pi \le x \le \pi$

The range of H is $0 \le y \le 1$

1975 AB1
Solution

(a) $f(-x) = \ln((-x)^2 - 9) = \ln(x^2 - 9) = f(x)$
Therefore the graph of f is symmetric with respect to the y-axis.

(b) Since we need $x^2 - 9 > 0$, the domain of f is the set $\{x \mid x < -3 \text{ or } x > 3\}$

(c) $f(x) = 0$ when $x^2 - 9 = 1$. This happens for $x = \pm\sqrt{10}$.

(d) Method 1:

$f(x) = \ln(x^2 - 9) \Rightarrow x^2 - 9 = e^{f(x)} = e^y$
Since $x > 3$, $x = \sqrt{e^y + 9}$.

Hence $f^{-1}(x) = \sqrt{e^x + 9}$.

Method 2:

$y = \ln(x^2 - 9)$, so interchanging variables gives $x = \ln(y^2 - 9)$.
$e^x = y^2 - 9$
$y = \sqrt{e^x + 9}$

Hence $f^{-1}(x) = \sqrt{e^x + 9}$.

1977 AB1
Solution

(a) $S(x) = g(f(x)) = \ln(\cos x)$

The domain of S is all x in the domain of f for which $\cos x > 0$, that is, $0 \le x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x \le 2\pi$.

(b) The range of S is $y \le 0$.

(c) $\ln(\cos x) = 0$

$\cos x = e^0 = 1$

The zeros are $x = 0$ and $x = 2\pi$.

1981 AB4

Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$.

(a) Is f an even or odd function? Justify your answer.

(b) Find the domain of f.

(c) Find the range of f.

1988 AB1
Solution

(a) $x^4 - 16x^2 \ge 0$

$x^2(x^2 - 16) \ge 0$

$x^2 \ge 16$ or $x = 0$

The domain of f is all x satisfying $|x| \ge 4$ or $x = 0$.

(b) The graph of f is symmetric about the y-axis because $f(-x) = f(x)$.

1989 AB4
Solution

(a) $x < -2$ or $x > 2$
or $|x| > 2$

(b) $x = 2,\ x = -2$

(c) $\lim_{x \to \infty} \frac{x}{\sqrt{x^2 - 4}} = 1$

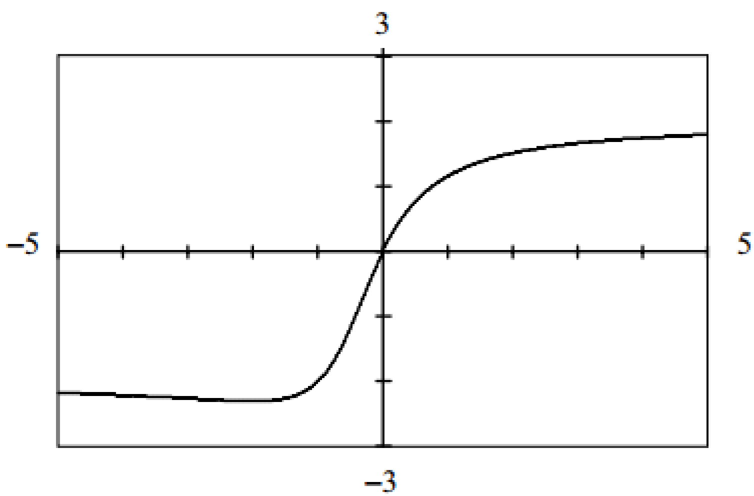
$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - 4}} = -1$

$y = 1,\ y = -1$

1995 AB1
Solution

(a) Domain: all real numbers since $x^2 + x + 1 > 0$

(b)



Viewing Window
$[-5, 5] \times [-3, 3]$

(c) $y = 2$ and $y = -2$