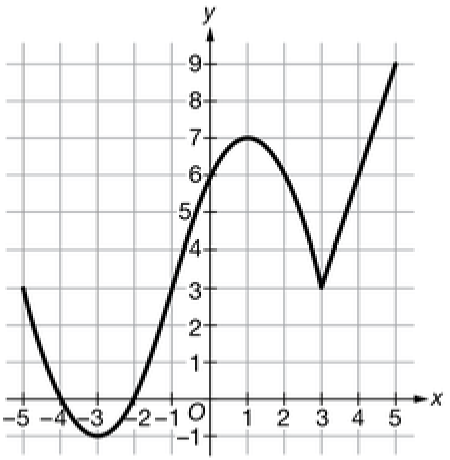
Unit 2 Progress Check: FRQ Part B

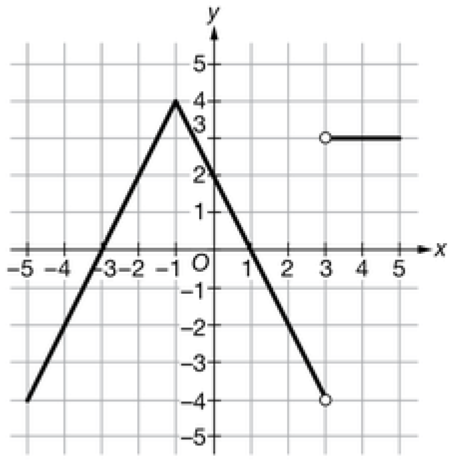
1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.





The graphs of the function f and its derivative f' are shown above for $-5 \leq x \leq 5$.

Find the average rate of change of f over the interval $-5 \leq x \leq 5$. For how many values of x in the interval $-5 \leq x \leq 5$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

Please respond on separate paper, following directions from your teacher.

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Write an equation for the line tangent to the graph of f at $x = 2$.

Please respond on separate paper, following directions from your teacher.

For each of $\lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$ and $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$, find the value or give a reason why it does not exist.

Please respond on separate paper, following directions from your teacher.

Let g be the function defined by $g(x) = f(x) \cdot \ln x$. Find $g'(4)$.

Please respond on separate paper, following directions from your teacher.

Part A

If the average rate of change has a maximum of one sign error, the response is eligible for the second point based on a correct interpretation of the graph of f' with the presented average rate of change.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ average rate of change
- ☐ answer

Solution:

$$\frac{f(5) - f(-5)}{5 - (-5)} = \frac{9 - 3}{5 - (-5)} = \frac{6}{10} = \frac{3}{5}$$

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The average rate of change of f over the interval $-5 \leq x \leq 5$ is $\frac{3}{5}$.

$f'(x)$, the instantaneous rate of change of f at x, equals $\frac{3}{5}$ for two values of x in the interval $-5 \leq x \leq 5$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The student response accurately includes a correct tangent line equation.

Solution:

$f(2) = 6$

$f'(2) = -2$

An equation of the line tangent to the graph of f at $x = 2$ is $y = 6 - 2(x - 2)$.

Part C

The second point is earned for “does not exist” with a reason that indicates f is not differentiable at that point (e.g., indicates graph has a corner point or sharp point).

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- ☐ $\lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$
- ☐

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$\lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$ does not exist

Solution:

$\lim_{x \to -1} \frac{f(x)-f(-1)}{x-(-1)} = f'(-1) = 4$

$\lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$ does not exist because f is not differentiable at $x = 3$.

Part D

The first point is earned for a correct application of the product rule. The second point is earned for a correct expression that substitutes both correct numerical values.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The student response accurately includes both of the criteria below.

- ☐ $g'(x)$
- ☐ answer

Solution:

$g'(x) = f'(x) \cdot \ln x + f(x) \cdot \frac{1}{x}$

$g'(4) = f'(4) \cdot \ln 4 + f(4) \cdot \frac{1}{4} = 3\ln 4 + 6 \cdot \frac{1}{4} = 3\ln 4 + \frac{3}{2}$

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2. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

t (minutes)	0	15	45	70	90	100
$A(t)$ (automobiles)	0	30	190	250	405	600

Prior to a sporting event, the number of automobiles that have entered a stadium parking lot is modeled by the differentiable function A, where t is the number of minutes since the parking lot opened. Values of $A(t)$ for selected values of t are given in the table above.

According to the model, what is the average rate at which automobiles enter the parking lot, in automobiles per minute, over the time interval $45 \leq t \leq 90$ minutes?

Please respond on separate paper, following directions from your teacher.

Write $A'(95)$ as the limit of a difference quotient. Use the data in the table to approximate $A'(95)$. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

What is the shortest time interval during which it is guaranteed that $A(t) = 400$ for some time t in the interval? Justify your answer.

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Please respond on separate paper, following directions from your teacher.

For $0 \leq t \leq 45$, the function f defined by $f(t) = \frac{2}{27}t^2 + \frac{8}{9}t$ models the number of automobiles that have entered the parking lot, where t is the number of minutes since the parking lot opened. Find $f'(10)$, the rate at which automobiles enter the parking lot, in automobiles per minute, at time $t = 10$ minutes.

Please respond on separate paper, following directions from your teacher.

Part A

The point is earned for a correct expression that substitutes both correct values from the table, with or without simplification.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The student response accurately includes a correct average rate of change.

Solution:

$$\frac{A(90)-A(45)}{90-45} = \frac{405-190}{90-45} = \frac{215}{45} = \frac{43}{9}$$

The average rate at which automobiles enter the parking the time interval $45 \leq t \leq 90$ minutes is $\frac{43}{9}$ automobiles per minute.

Part B

The second point is earned for a correct expression that substitutes both correct values from the table, with or without simplification.

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ limit expression
- ☐ approximation

Solution:

$A'(95) = \lim_{t\to 95} \frac{A(t)-A(95)}{t-95}$

OR $A'(95) = \lim_{h\to 0} \frac{A(95+h)-A(95)}{h}$

$A'(95) \approx \frac{A(100)-A(90)}{100-90} = \frac{600-405}{100-90} = \frac{195}{10} = \frac{39}{2}$

Part C

1 out of 2 points if correct justification does not specifically reference IVT.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ time interval
- ☐ justification, using Intermediate Value Theorem

Solution:

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The shortest time interval for which it is guaranteed that $A(t) = 400$ is $70 \leq t \leq 90$.

Because A , is differentiable, A , is continuous.

$A(70) = 250 < 400 < 405 = A(90)$

By the Intermediate Value Theorem, there must be at least one time t, for $70 \leq t \leq 90$, such that $A(t) = 400$.

Part D

The first point is earned for demonstration of a correct derivative for f and does not require a separate statement of $f'(t)$. The derivative and evaluation of the derivative can be expressed in one line to earn both points.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ $f'(t)$
- ☐ $f'(10)$

Solution:

$f'(t) = \frac{4}{27}t + \frac{8}{9}$

$f'(10) = \frac{4}{27} \cdot 10 + \frac{8}{9} = \frac{40}{27} + \frac{24}{27} = \frac{64}{27}$

At time $t = 10$ minutes, automobiles enter the parking lot at a rate of $\frac{64}{27}$ automobiles per minute.