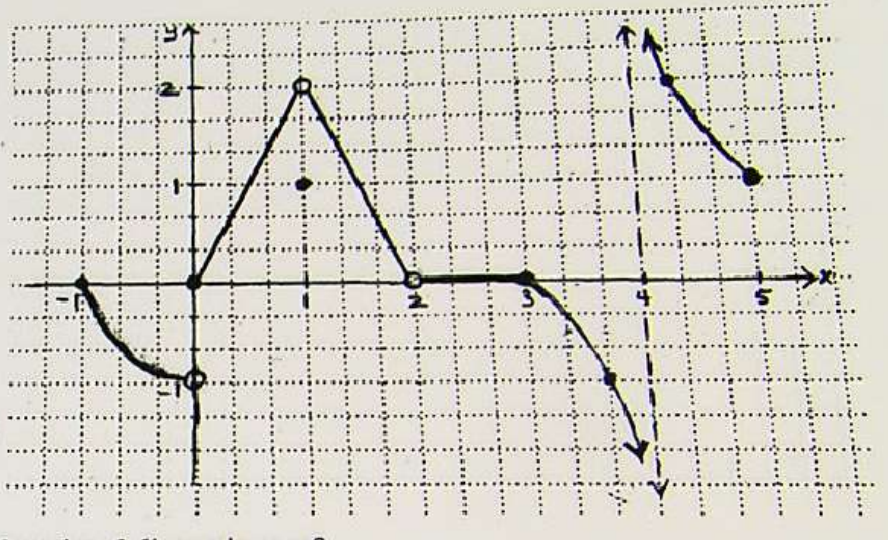


What makes a function continuous?

The Meaning of Continuity:

Consider the function f defined below:

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 \leq x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \leq 3 \\ \frac{1}{2(x-4)} + 0.5, & 3 < x \leq 5, x \neq 4 \end{cases}$$



- Over the interval $[-1, 5]$, where is the function f discontinuous?
- Complete the table for each value of $x = c$ given.

$X = c$	$f(c)$	$\lim_{x \rightarrow c^-} f(x)$	$\lim_{x \rightarrow c^+} f(x)$	$\lim_{x \rightarrow c} f(x)$	Does $f(c) = \lim_{x \rightarrow c} f(x)$	Is $f(x)$ continuous at $x = c$?
0						
1						
2						
3						
4						

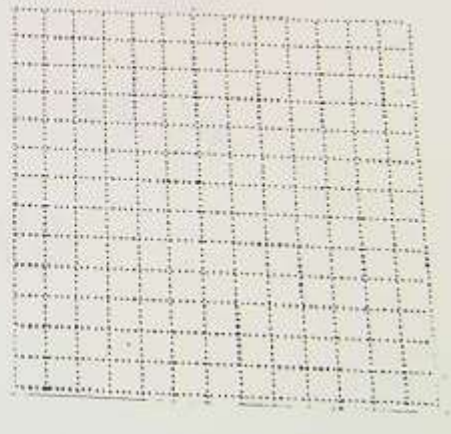
Using the table above, determine the three conditions that must exist in order for a function to be continuous:

- The $\lim_{x \rightarrow c} f(x)$ _____
- $f(c)$ _____
- $\lim_{x \rightarrow c} f(x)$ _____ $f(c)$

EXAMPLE: Let f be the function defined by $f(x) = \begin{cases} x+1, & x < 2 \\ k(x-5)^2, & x \geq 2 \end{cases}$

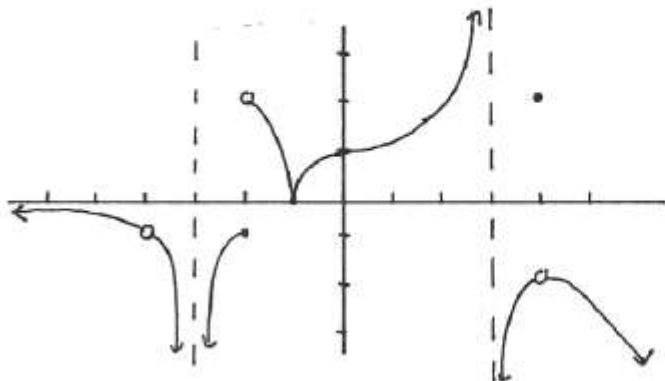
where k stands for a constant.

- Graph this function for $k = 1$.
- Use the definition of continuity to explain formally what must be true in order for f to be continuous at $x = 2$.
- Find the value of k that makes f continuous at $x = 2$. Then sketch f for this value of k .



6. Use the graph below.

- Find all x values where the graph is discontinuous.
- For each discontinuous x value, name the type of discontinuity.
- For each discontinuous x value, state why it fails the formal definition of continuity.



$$7. f(x) = \begin{cases} \frac{x^2 + 5x + 6}{x^2 - 4}, & \text{if } x \neq \pm 2 \\ -\frac{1}{4}, & \text{if } x = \pm 2 \end{cases}$$

Where is $f(x)$ discontinuous? (A) Nowhere (B) $x = 2$ (C) $x = -2$ (D) $x = \pm 2$ (E) \mathbb{R}

Find the value of a and b so that $f(x)$ is continuous

$$8. f(x) = \begin{cases} ax - 1, & x < -1 \\ -x^2 + 1, & -1 \leq x < 2 \\ \frac{1}{2}x + b, & x \geq 2 \end{cases}$$

$$9. f(x) = \begin{cases} 2x + a, & x \leq -1 \\ x^2 + 1, & -1 < x \leq 2 \\ bx - 1, & x > 2 \end{cases}$$

$$10. f(x) = \begin{cases} \frac{x^2 + 7x + 10}{x + 2} & \text{if } x \neq -2 \\ b & \text{if } x = -2 \end{cases}$$

Are the following continuous at $x = 2$? If not, give the official reason according to the formal definition of continuity.

$$11. f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$

$$12. f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$

$$13. f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 3x + 2 & \text{if } x > 2 \end{cases}$$

$$14. f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$