Key

4. (1998 BC 26) The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

E

$\frac{dP}{dt} = 2P\left(1 - \frac{P}{10{,}000}\right)$

A. 2,500 B. 3,000 C. 4,200 D. 5,000 (E.) 10,000

5. Find the carrying capacity for a population growth rate modeled by $\frac{dP}{dt} = 6P - 0.012P^2$.

$\frac{dP}{dt} = 6P - \frac{12P^2}{1000} = 6P - \frac{6P^2}{500}$

$\frac{dP}{dt} = 6P\left(1 - \frac{P}{500}\right)$

A

(a.) 500 b. 50 c. 0.012 d. 0.002 e. None of these

A population of rabbits in a certain habitat grows according to the differential equation $\frac{dy}{dt} = y\left(1 - \frac{1}{10}y\right)$, where t is measured in months ($t \geq 0$) and y is measured in hundreds of rabbits per months. There were initially 100 rabbits in this habitat that is $y(0) = 1$

10. What is the fastest growth rate, in rabbits per month that this population exhibits? *at half of carrying capacity*

a. 50
b. 100
c. 200
(d.) 250
e. 500

D

$M = 10$

$\frac{dy}{dt} = 5\left(1 - \frac{5}{10}\right) = 5\left(\frac{5}{10}\right) = \frac{25}{10} = 2.5 \times 100 = 250$

(11.) Estimates of y(t) can be produced using Euler's Method with step size $\Delta t = 1$. To the nearest rabbit, the estimate for y(2) is

Calc ok

a. 281
b. 300
(c.) 344
d. 379
e. 500

C

t	y	dy/dt
0	1	.9
1	1.9	1.539
2	3.439	

$y = 1(.9) + 1$

$y = 1(1.539) + 1.9$

$\approx 3.44 \times 100$

$= 344$

Water flows continuously from a large tank at a rate proportional to the amount of water remaining in the tank; that is $\frac{dy}{dt} = ky$. There was initially 10,000 cubic feet of water in the tank, and at the time t = 4 hours, 8000 cubic feet remained. You may use a calculator on these two problems.

$t = 0,\ y = 10{,}000$

$t = 4,\ y = 8000$

13. What is the value of k in the equation $\frac{dy}{dt} = ky$?

C

a. -0.050
b. 0.169
(c.) -0.056
d. -0.200
e. -0.223

$y = Ce^{kt}$

$10{,}000 = Ce^0$

$C = 10{,}000$

$8000 = 10{,}000e^{k \cdot 4}$

$K = -.056$

STORE

14. To the nearest cubic foot, how much water remained in the tank at time = 8 hours?

a. 5778
(b.) 6400
c. 6458
d. 6619
e. 6000

B

$y = 10{,}000e^{-.056(8)}$

$y = 6389.048$

6389