

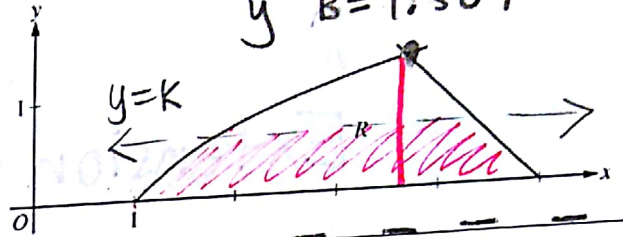
Calculator ACTIVE

Name: \_\_\_\_\_

Points Earned	14	13	12/11	10/9	8/7	6	5	4	3/2/1/0
Grade	100	94	89	84	79	74	69	60	45

Let R be the region in the first quadrant bounded by the x-axis and the graphs of  $y = \ln x$  and  $y = 5 - x$ , as shown in the figure below.

$x \quad A = 3.693$   
 $y \quad B = 1.307$



$x = -y - 5 \quad x = 5 - y$   
 $x = -y + 5 \quad x = e^y$

a) Find the area of R.

$$\int_1^A (\ln x) dx + \int_A^5 (5-x) dx$$

OR  $\int_0^B (5-y - e^y) dy$

- ① integrand
- ① limits
- ① answer

2.985 or 2.986

b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is square.

$$\int_1^A (\ln x)^2 dx + \int_A^5 (5-x)^2 dx$$

- ① integrands
- ① limits
- ① answer

2.784

c) The horizontal line  $y = k$  divides R into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value k.

$$\int_0^k (5-y - e^y) dy = \frac{1}{2} (2.986)$$

1.493

- ① integrand
- ① limits
- ① equation

## Extension Questions:

d)

$$A = \frac{1}{2} \pi r^2 \quad d = (\ln x) \quad d = 5 - x$$

$$A = \frac{1}{2} \pi \left( \frac{(\ln x)}{2} \right)^2 \quad r = \frac{(\ln x)}{2} \quad r = \frac{5-x}{2}$$

$$A = \frac{1}{2} \pi \left( \frac{5-x}{2} \right)^2$$

$$\frac{\pi}{8} \int_1^A (\ln x)^2 dx + \frac{\pi}{8} \int_A^5 (5-x)^2 dx$$

$$\frac{\pi}{8} (2.041) + \frac{\pi}{8} (.743) = \frac{\pi}{8} (2.784)$$

$$.801 + .292$$

$$1.093$$

e) around  $y = -1$

$$\pi \cdot \int_1^A \left[ (\ln x - (-1))^2 - (0 - (-1))^2 \right] dx + \pi \int_A^5 \left[ (5-x - (-1))^2 + (0 - (-1))^2 \right] dx$$