1. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

2. If the function f is defined by $f(x) = x^5 - 1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x) =$

(A) $\frac{1}{\sqrt[5]{x}+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

(D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

3. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

4. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

5. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

6. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

7. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

8. $\int_0^1 (x+1)e^{x^2+2x}\, dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

9. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}} dx =$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

10. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

11. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x + C$ (E) $5\ln\left(1+x^2\right)+C$

12. If $\frac{dy}{dx}=\cos(2x)$, then $y=$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

13. Which of the following is equal to $\ln 4$?

(A) $\ln 3+\ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\,dt$ (D) $\int_1^4 \ln x\,dx$ (E) $\int_1^4 \frac{1}{t}\,dt$

14. If $y=10^{\left(x^2-1\right)}$, then $\frac{dy}{dx}=$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

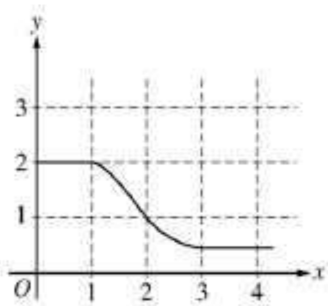
(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

15. If $y=\arctan(\cos x)$, then $\frac{dy}{dx}=$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2\sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$ (E) $\frac{1}{1+\cos^2 x}$

16.



The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x)=f(x)$, then $F(3)-F(0)=$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

ANSWERS: 1969 #4,14,23,29,33,35, 1973#8,21,27,30,32, 1985#4,7,10,20, 1997#78, FR from 2011 AB.

1. D	2. E	3. C	4. A	5. A	6. A	7. B	8. B
9. E	10. B	11. D	12. C	13. E	14. D	15. A	16. D

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(a) $H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$

$= \frac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right)$$
$= 52.95$

$3 : \begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

$2 : \begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$