Unit 5 Problem Set #2:

1 $y = \ln x - 1$

E $x = \ln y - 1$

$x+1 = \ln y$

$e^{x+1} = y$

2 $y = 6\ln(x^{-2})$

D $y' = 6 \cdot \frac{1}{x^{-2}} \cdot -2x^{-3}$

$y' = \frac{-12}{x}$

3 I. $\lim_{x\to 0} e^x \sin x = 1 \cdot 0 = 0$ T

E

II. $\lim_{x\to 0} e^x(\cos x + \sin x) = 1$ T

$1(1+0)$

$f'(x) = e^x \cdot \cos x + \sin x \cdot e^x$

$e^x(\cos x + \sin x)$

III. $\lim_{x\to 0} f''(x) = 2$ T

$f''(x) = e^x(-\sin x + \cos x) +$
$(\cos x + \sin x)e^x$

$1(0+1) + (1+0)\cdot 1$

$1 + 1 = 2$

4 $y = \ln(\tan x)$

A $y' = \frac{1}{\tan x} \cdot \sec^2 x$

$y' = \frac{1}{\cos^2 x \tan x}$

$y' = \frac{1}{\cos^2 x \cdot \frac{\sin x}{\cos x}}$

double angle ID

$y' = \left(\frac{1}{\cos x \sin x}\right) \cdot \frac{2}{2} = \frac{2}{\sin(2x)}$

5 $y = e^{5x+5}$

C $y' = 5e^{5x+5}$

$y'(0) = 5e^5$

6 $y = \ln e^{\tan^2 x}$

E $y = \tan^2 x = (\tan x)^2$

$y' = 2(\tan x)^1 \cdot \sec^2 x$

$y'(\frac{\pi}{4}) = 2(\tan \frac{\pi}{4}) \cdot \frac{1}{\cos^2(\frac{\pi}{4})}$

$y'(\frac{\pi}{4}) = 2 \cdot 1 \cdot \frac{1}{(\frac{\sqrt{2}}{2})^2}$

$= \frac{2}{\frac{1}{2}} = 4$

7 $y = \arctan(3x)$

C $y' = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$

8 $y = \arcsin(x^2)$

B $y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$

$y' = \frac{2x}{\sqrt{1-x^4}}$

9 $f(x) = \ln x$

B $f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = \frac{-1}{x^2}$

$f''(3) = \frac{-1}{9}$

10

C

$f(x) = \ln(3-2x)$

$f'(x) = \frac{1}{3-2x} \cdot -2$

$f'(1) = \frac{-2}{3-2} = -2$

perp slope $= \frac{1}{2}$

$(1, \ln 1) = (1, 0)$

$y = \frac{1}{2}(x-1)$

11

C

$y = e^x \sin x$

$y' = e^x \cos x + \sin x \cdot e^x$

12

C

$\frac{f(b)-f(a)}{b-a}$

$\frac{\ln 4 - \ln 1}{1} = \ln 4$

13

A

$y = 2 - \ln x$

No HA

14

D

$f(x) = \ln(\sin x)$

$f'(x) = \frac{1}{\sin x} \cdot \cos x$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

15

D

$y = \ln\left(e^{x^2-1}\right)$

$y' = \frac{1}{e^{x^2-1}} \cdot e^{x^2-1} \cdot 2x$

$y'(1) = \frac{2}{1} = 2$

17

B

$f(1,2)$

$f^{-1}(2,1)$

$(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{8}$

$f'(x) = 5x^4 + 3$

$f'(1) = 5 + 3 = 8$

18

A

$y = e^x - x + 2$

$y' = e^x - 1 = 0$

$e^x = 1$

$x = 0$

$y(0) = 1 - 0 + 2 = 3$

$f'(x)$: $-$ | $+$

x: 0

$(0, 3)$

19

B

$y = \ln(x^2+1)$

$y' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$y'' = \frac{(x^2+1)(2) - (2x)(2x)}{(x^2+1)^2} = 0$

$2x^2 + 2 - 4x^2 = 0$

$-2x^2 = -2$

$x^2 = 1$

$x = \pm 1$

$f''(x)$: $-$ | $+$ | $-$

x: -1, 1

cc↑: $(-1, 1)$

$y'' = \frac{-2x^2+2}{(x^2+1)^2}$

20

E

$y = 4^{x^2}$

$y' = 4^{x^2} \cdot \ln 4 \cdot 2x$

$y'(1) = 4 \cdot \ln 4 \cdot 2 = 8 \ln 4$

(21)

E $\displaystyle\int \frac{e^{2x}-e^{3x}}{e^{x}}\,dx$

$= \int (e^{x}-e^{2x})\,dx$

$= e^{x}-\frac{1}{2}e^{2x}+C$

(22)

A $\displaystyle\int_0^{\pi/2} e^{2-\cos x}\cdot \sin x\,dx$

$u = 2-\cos x$

$du = \sin x\,dx$

$\displaystyle\int_1^2 e^{u}\,du =$

$u(0) = 1$

$u(\pi/2) = 2$

$e^{u}\Big|_1^2 = e^2 - e$

(23) $y = \ln(\tan^2 x)$

D $y' = \dfrac{1}{\tan^2 x}\cdot 2\tan x\cdot \sec^2 x$

$y' = \dfrac{2}{\cos^2 x\cdot \tan x}$

$y'\left(\frac{\pi}{4}\right) = \dfrac{2}{\cos^2\left(\frac{\pi}{4}\right)\cdot \tan\frac{\pi}{4}}$

$= \dfrac{2}{\left(\frac{\sqrt{2}}{2}\right)^2\cdot 1} = \dfrac{2}{\frac{1}{2}\cdot 1} = 4$

(24) $y = e^{\sin^2 x}$

C $y' = e^{\sin^2 x}\cdot 2\sin x\cdot \cos x$

(25) $v(t) = \ln(t^2+t+1)$

C $a(t) = \dfrac{1}{t^2+t+1}\cdot 2t+1$

$a(1) = \dfrac{1}{3}\cdot 3 = 1$

(26) $y = \dfrac{e^{2x-1}}{x}$

E $y' = \dfrac{x(e^{2x-1}\cdot 2) - e^{2x-1}\cdot 1}{x^2}$

$y' = \dfrac{e^{2x-1}(2x-1)}{x^2} = 0 \qquad x = \frac{1}{2}$

$f'(x)$ $-$ $+$

x $\frac{1}{2}$

graph: HA $y=0$

VA $x=0$

(27) $y = \ln(4x+1)$

D $y' = \dfrac{4}{4x+1}$

$y'' = \dfrac{(4x+1)(0)-(4)(4)}{(4x+1)^2}$

$y'' = \dfrac{-16}{(4x+1)^2}$

(28)

D $\displaystyle\int \frac{1}{1+(2x)^2}\,dx$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$a = 1$

$u = 2x$

$\displaystyle\frac{1}{2}\int \frac{1}{1+u^2}\,du$

$\frac{1}{2}\tan^{-1}(2x) + C$

29 $y = \arcsin(\frac{1}{2}x)$

A $y' = \frac{1}{\sqrt{1-(\frac{1}{2}x)^2}} \cdot \frac{1}{2}$

$y' = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$

30 $\int \frac{1}{x^2+2x+2}\,dx$ $\quad x^2+2x+\boxed{1}+2-\boxed{1}$

C $\quad (x+1)^2+1$

$\int \frac{1}{(x+1)^2+1}\,dx$

$a=1$

$u=x+1$

$= \arctan(x+1)+C$