

Problem Set #1 Unit 5

① $\int_0^8 \frac{1}{\sqrt{1+x}} dx$ $u = 1+x$
 D $du = dx$
 $\int_1^9 u^{-1/2} du$ $u(0) = 1$
 $u(8) = 9$
 $2u^{1/2} \Big|_1^9 = 2\sqrt{9} - 2\sqrt{1}$
 $\frac{6-2}{4}$

② $y = x^5 - 1$
 E $x = y^5 - 1$
 $x + 1 = y^5$
 $y = \sqrt[5]{x+1}$

③ $\int_0^2 e^{2x} dx$ $u = 2x$
 C $du = 2dx$
 $\frac{1}{2} du = dx$
 $\frac{1}{2} \int_0^4 e^u du$ $u(0) = 0$
 $u(2) = 4$
 $\frac{1}{2} (e^u \Big|_0^4) = \frac{1}{2} (e^4 - e^0)$
 $\frac{1}{2} (e^4 - 1)$

④ $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx$ $u = \sin x$
 A $du = \cos x dx$
 $u(\pi/4) = \sqrt{2}/2$
 $u(\pi/2) = 1$
 $\int_{\sqrt{2}/2}^1 \frac{1}{u} du = \ln|u| \Big|_{\sqrt{2}/2}^1$
 $\ln 1 - \ln(\sqrt{2}/2)$
 $-\ln(\sqrt{2}/2)$
 $\ln(\frac{\sqrt{2}}{2})^{-1}$
 $\ln(\frac{2}{\sqrt{2}}) = \ln \sqrt{2}$

⑤ $\int_{-1}^2 (3t^3 - t^2) dt$
 A $\frac{-1}{2 - (-1)}$

$\frac{\frac{3}{4}t^4 - \frac{1}{3}t^3 \Big|_{-1}^2}{3}$
 $\left[\frac{3}{4}(2)^4 - \frac{1}{3}(2)^3 \right] - \left[\frac{3}{4} + \frac{1}{3} \right]$
 $\frac{(12 - \frac{8}{3}) - \frac{13}{12}}{3} = \frac{11}{4}$

⑥ $v(t) = \int 24t^2 dt$
 A $v(t) = 8t^3 + C$
 $0 = 0 + C$
 $C = 0$

$v(t) = 8t^3$
 $\int_0^2 18t^3 dt = 32$
 Calc

⑦ $v(t) = t^2$

B $\int_1^2 t^2 dt = \frac{t^3}{3} \Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

⑧ $\int_0^1 (x+1)e^{x^2+2x} dx$ $u = x^2 + 2x$
 B $du = (2x+2)dx$
 $\frac{1}{2} du = (x+1)dx$
 $u(0) = 0$
 $u(1) = 3$
 $\frac{1}{2} \int_0^3 e^u du = \frac{1}{2} e^u \Big|_0^3$
 $= \frac{1}{2} (e^3 - 1)$

⑨ $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}} dx$ $u=1-x^2$
 E $du = -2x dx$
 $u(0) = 1$

$-\int u^{-1/2} du = -2u^{1/2}$ $u(\frac{1}{2}) = \frac{3}{4}$

$-2(1-x^2)^{1/2}$
 $-2\sqrt{1-x^2} \Big|_0^{\frac{1}{2}}$

$-2\left[\sqrt{1-\frac{1}{4}} - \sqrt{1-0}\right]$

$-2\left[\sqrt{\frac{3}{4}} - 1\right]$

$-2\left(\frac{\sqrt{3}}{2} - 1\right)$

$-\sqrt{3} + 2$

⑩

B $\int_1^2 \frac{x-4}{x^2} dx$

$\int_1^2 \left(\frac{1}{x} - 4x^{-2}\right) dx$

$= \ln|x| + \frac{4}{x} \Big|_1^2$

$= (\ln 2 + 2) - (0 + 4)$

$\ln 2 - 2$

⑪

D $\int \frac{5}{1+x^2} dx$

$= 5 \int \frac{1}{1+x^2} dx$

$= 5 \arctan x + C$

⑫

C $\frac{dy}{dx} = \cos(2x)$

$\int dy = \int \cos(2x) dx$ $u=2x$
 $du = 2dx$
 $\frac{1}{2} du = dx$

$y = \frac{1}{2} \int \cos u du$

$y = \frac{1}{2} \sin(2x) + C$

⑬

E $\int_1^4 \frac{1}{t} dt = \ln|t| \Big|_1^4$

$\ln 4 - \ln 1$

$\ln\left(\frac{4}{1}\right) = \ln 4$

⑭

D $y = 10^{x^2-1}$

$\frac{dy}{dx} = 10^{x^2-1} \cdot \ln 10 \cdot 2x$

⑮

A $y = \arctan(\cos x)$

$\frac{dy}{dx} = \frac{1}{1+(\cos^2 x)} \cdot -\sin x$

⑯

D $F(3) = F(0) + \int_0^3 f(x) dx$

$F(3) - F(0) = \int_0^1 f(x) dx + \int_1^3 f(x) dx$

$F(3) - F(0) = 2 + 2 \cdot 3$

$4 \cdot 3 = 14$