

Name: Key

Calculator Inactive

1) E    2) A    3) B    4) C    5) B

<p>1.</p> <p>E</p>	<p>What are all horizontal asymptotes of the graph of <math>y = \frac{5+2^x}{1-2^x}</math> in the <math>xy</math>-plane?</p> <p>(A) <math>y = -1</math> only            (B) <math>y = 0</math> only            (C) <math>y = 5</math> only            (D) <math>y = -1</math> and <math>y = 0</math>            (E) <math>y = -1</math> and <math>y = 5</math></p> <p><math>\lim_{x \rightarrow \infty} \frac{5+2^x}{1-2^x}</math> L'H: <math>\frac{2^x \cdot \ln 2}{-2^x \ln 2} = -1</math></p> <p><math>\lim_{x \rightarrow -\infty} \frac{5+2^x}{1-2^x} = \frac{5 + (\frac{1}{2^x})^0}{1 - (\frac{1}{2^x})^0} = \frac{5}{1} = 5</math></p>
<p>2.</p> <p>A</p>	<p>What is the slope of the line tangent to the curve <math>y = \arctan(4x)</math> at the point at which <math>x = \frac{1}{4}</math>?</p> <p>(A) 2    (B) <math>\frac{1}{2}</math>    (C) 0    (D) <math>-\frac{1}{2}</math>    (E) -2</p> <p><math>\frac{1}{1+(4x)^2} \cdot 4 = \frac{4}{1+16x^2} = \frac{4}{1+16(\frac{1}{4})^2} = \frac{4}{1+16(\frac{1}{16})} = \frac{4}{2} = 2</math></p>
<p>3.</p> <p>B</p>	<p>The function <math>f(x) = x^5 + 3x - 2</math> passes through the point <math>(1, 2)</math>. Let <math>f^{-1}</math> denote the inverse of <math>f</math>. Then <math>(f^{-1})'(2)</math> equals</p> <p>a) <math>1/83</math>    (b) <math>1/8</math>    c) 1    d) 8    e) 83</p> <p><math>\frac{1}{f'(1)} = \frac{1}{8}</math></p> <p><math>f(1, 2)</math>  <math>f^{-1}(2, 1)</math>  <math>f'(x) = 5x^4 + 3</math>  <math>f'(1) = 5 + 3 = 8</math></p>
<p>4.</p> <p>C</p>	<p><math>\int \frac{dx}{x^2 + 2x + 2} =</math></p> <p>a. <math>\ln(x^2 + 2x + 2) + C</math>            b. <math>\ln x+1  + C</math>            c. <math>\arctan(x+1) + 3</math>            d. <math>\frac{1}{\frac{1}{3}x^3 + x^2 + 2x} + C</math>            e. <math>-\frac{1}{x} + \frac{1}{2} \ln x  + \frac{x}{2} + C</math></p> <p><math>x^2 + 2x + \square + 2 - \square</math>  <math>(x+1)^2 + 1</math>  <math>\int \frac{1}{(x+1)^2 + 1} dx</math></p>
<p>5.</p> <p>B</p>	<p><math>\lim_{x \rightarrow 0} \frac{\int_1^{1+x} \frac{\cos t}{t} dt}{x}</math> is</p> <p>A -cos 1            B cos 1            C -sin 1</p> <p>L'H: <math>\frac{\cos(1+x)}{1+x}</math>  <math>\cos 1</math>            D sin 1            E Does not exist</p>

Let  $f$  be the function defined by  $f(x) = e^x \cos x$  with domain  $[0, 2\pi]$ .

a) Find the absolute minimum value of  $f(x)$

$$f'(x) = e^x(-\sin x) + \cos x \cdot e^x$$

$$f'(x) = e^x(\cos x - \sin x) = 0$$

$$\cos x = \sin x$$

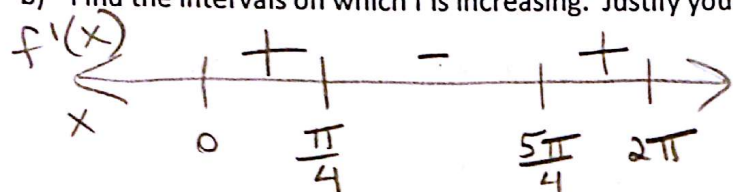
$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

(1)

$$\text{Abs min value} = -\frac{\sqrt{2}}{2} e^{5\pi/4}$$

x	y (1)
0	1
$\pi/4$	$e^{\pi/4} \cdot \frac{\sqrt{2}}{2}$
$5\pi/4$	$e^{5\pi/4} \cdot (-\frac{\sqrt{2}}{2})$
$2\pi$	$e^{2\pi} \cdot 1$

b) Find the intervals on which  $f$  is increasing. Justify your answer



Inc:  $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$   
 b/c  $f'(x)$  is pos (1)

$$e^{\pi/6}(\cos \frac{\pi}{6} - \sin \frac{\pi}{6})$$

$$e^{\pi/6}(\frac{\sqrt{3}}{2} - \frac{1}{2})$$

$$e^{\pi/2}(\cos \frac{\pi}{2} - \sin \frac{\pi}{2})$$

$$e^{\pi/2}(0 - 1)$$

$$e^{7\pi/4}(\cos 7\pi/4 - \sin 7\pi/4)$$

$$e^{7\pi/4}(\frac{\sqrt{2}}{2} - (-\frac{\sqrt{2}}{2}))$$

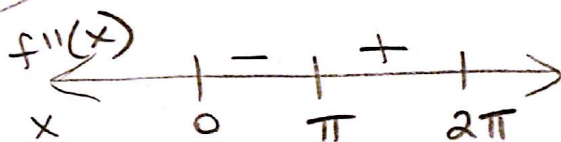
c) Find the x-coordinate of each point of inflection on the graph

$$f''(x) = e^x(-\sin x - \cos x) + (\cos x - \sin x)e^x$$

$$f''(x) = -e^x \sin x - e^x \cos x + e^x \cos x - e^x \sin x$$

$$f''(x) = -2e^x \sin x = 0$$

$$x = 0, \pi, 2\pi$$



POI:  $x = \pi$  (1)

d) Estimate the function when  $x = (\frac{\pi}{2} + .1)$  using the tangent line approximation at  $\frac{\pi}{2}$ . Is your estimate greater than or less than the true value? Give a reason for your answer.

$$f(\frac{\pi}{2}) = e^{\pi/2} \cdot \cos \frac{\pi}{2} = 0$$

$$f'(\frac{\pi}{2}) = e^{\pi/2} \cdot (\cos \frac{\pi}{2} - \sin \frac{\pi}{2}) = e^{\pi/2}(-1) = -e^{\pi/2}$$

$$y = -e^{\pi/2}(x - \frac{\pi}{2}) \quad (1)$$

$$(1) y(\frac{\pi}{2} + .1) = -e^{\pi/2}(\frac{\pi}{2} + .1 - \frac{\pi}{2})$$

$$= -e^{\pi/2}(.1)$$

$$= -.1 e^{\pi/2}$$

At  $x = \frac{\pi}{2}$ , (1)  
 graph is CC↓  
 so estimate is greater than actual