B

A polar curve is given by $r = \dfrac{3}{2-\cos\theta}$. The slope of the curve at $\theta = \dfrac{\pi}{2}$ is $\dfrac{dy}{dx} = \dfrac{\frac{3}{2}\cdot 0 + 1\left(-\frac{3}{4}\right)}{-\frac{3}{2}\cdot 1 + 0\left(-\frac{3}{4}\right)} = \dfrac{1}{2}$

A) 0 B) 0.5 C) 0.75 D) -0.75 E) Not defined.

$r' = \dfrac{-3\sin\theta}{(2-\cos\theta)^2}$ $r\left(\frac{\pi}{2}\right) = \frac{3}{2}$ $r'\left(\frac{\pi}{2}\right) = -\frac{3}{4}$ $\cos\frac{\pi}{2} = 0$ $\sin\frac{\pi}{2} = 1$

B

At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

$a(t) = \langle 2t, 5 \rangle$

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

$a(3) = \langle 6, 5 \rangle$

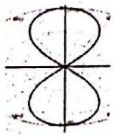
D

What is the area enclosed by the lemniscate $r^2 = -25\cos 2\theta$?

25/8 B) 25/4 C) 25/2 D) 25 E) 50

$0 = -25\cos 2\theta$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$



$2 \cdot \frac{1}{2} \cdot -25 \int_{\pi/4}^{3\pi/4} (\cos 2\theta)\, d\theta$

$= -25 \cdot \frac{1}{2} \sin 2\theta \Big|_{\pi/4}^{3\pi/4}$

$-\frac{25}{2}\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right)$

$= 25$

C

Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

$\int_0^{\pi} \left[(\cos(t^3)\cdot 3t^2)^2 + (5e^{5t})^2\right] dt$

(A) $\int_0^{\pi} \sqrt{\sin^2(t^3) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4\cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2\cos(t^3) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

D

In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

$\left.\frac{dy}{dx}\right|_{x=2} = 2x - 1 = 3$

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2\sqrt{10}$

$\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 3$ $\frac{dx}{dt} = \boxed{\dfrac{\frac{dy}{dt}}{3}}$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 40$

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$$\left(\frac{\frac{dy}{dt}}{3}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 40$$

Let $b = \frac{dy}{dt}$

$$\frac{b^2}{9} + b^2 = 40$$

$$b^2 + 9b^2 = 360$$

$$10b^2 = 360$$

$$b^2 = 36$$

$$b = 6$$

so $\frac{dy}{dt} = 6$

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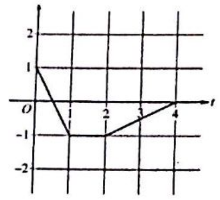
Calculator Active



At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

a) Find the position of the particle at t = 3

$$x(3) = x(0) + \int_0^3 (t^2 + \sin(3t^2))\,dt$$

$x(3) = 14.377$

$y(3) = -\frac{1}{2}$

position: $(14.377, -\frac{1}{2})$

(1) integral
(1) initial condition
(1) answer

b) Find the slope of the line tangent to the path of the particle at t = 3.

$$\frac{dy}{dx} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=3} = \frac{\frac{1}{2}}{9.956} = .050 \text{ or } .05$$

(1) answer

c) Find the speed of the particle at t = 3

$$\sqrt{(x'(3))^2 + (y'(3))^2}$$

$$\left.\sqrt{(t^2 + \sin(3t^2))^2 + (\tfrac{1}{2})^2}\,\right|_{t=3} = 9.969 \text{ or } 9.968$$

(1) expression for speed
(1) answer

d) Find the total distance traveled by the particle from t = 0 to t = 2

$$\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\,dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\,dt$$

$= 4.350$ or 4.349

4.35

(1) expression for dist.
(1) integrals
(1) answer

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