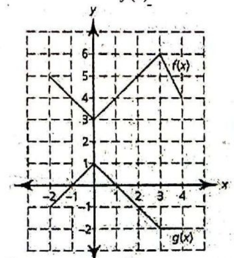
Key

Problem Set #2 – Derivatives Show all work on a separate sheet of paper
Name: ______________________ Date: ______________

1. D	What is the value of k + c if $f(x)=\begin{cases}2kx^2-x, & x>3\\ x^3+cx, & x\le 3\end{cases}$ is everywhere differentiable? a. 5/4 b. 3 c. 8 d. 11 e. 24
2. C	The graphs of f(x) and g(x) are shown below. If $h(x)=\frac{g(2x)}{f(x)}$, use the graphs to find h'(1).  a) -7/4 b) -9/16 c) 7/16 d) -5/16 e) -3/16
3. E	Let f and g be differentiable functions with the following properties: (i) g(x) > 0 for all x (ii) f(0) = 1 If $h(x)=f(x)g(x)$ and $h'(x)=f(x)g'(x)$, then f(x) = (A) f'(x) (B) g(x) (C) e^x (D) 0 (E) 1

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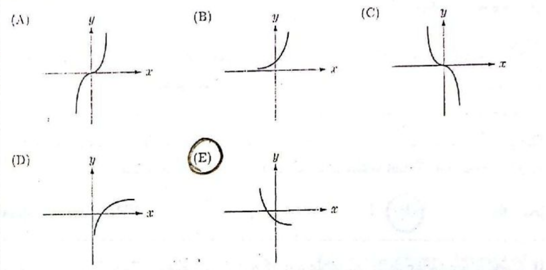
4. E	If, for all real numbers x, $f'(x)<0$ and $f''(x)>0$, which of the following curves could be part of the graph f? 
5. B	What is $\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^8-8\left(\frac{1}{2}\right)^8}{h}$? (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist. (E) It cannot be determined from the information given.
6. B	The slope of the line tangent to the graph of $y=\ln\left(x^2\right)$ at $x=e^2$ is (A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$
7. D	If $f(x)=\frac{x-1}{x+1}$ for all $x\ne -1$, then $f'(1)=$ (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

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8. If $y=\cos^2 3x$, then $\frac{dy}{dx}=$

A

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$
(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

9. The position of a particle moving along a straight line at any time t is given by $s(t)=t^2+4t+4$. What is the acceleration of the particle when $t=4$?

B

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

10. If f and g are twice differentiable and if $h(x)=f(g(x))$, then $h''(x)=$

A

(A) $f''(g(x))[g'(x)]^2+f'(g(x))g''(x)$
(B) $f''(g(x))g'(x)+f'(g(x))g''(x)$
(C) $f''(g(x))[g'(x)]^2$
(D) $f''(g(x))g''(x)$
(E) $f''(g(x))$

11. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2)=-5$, $f(5)=5$, and $f(9)=-5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2<c<5$, $f(c)=3$.

E

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

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12. An equation of the line tangent to the graph of $y=\cos(2x)$ at $x=\frac{\pi}{4}$ is

E

(A) $y-1=-\left(x-\frac{\pi}{4}\right)$
(B) $y-1=-2\left(x-\frac{\pi}{4}\right)$
(C) $y=2\left(x-\frac{\pi}{4}\right)$
(D) $y=-\left(x-\frac{\pi}{4}\right)$
(E) $y=-2\left(x-\frac{\pi}{4}\right)$

13. At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

B

(A) $\left(\frac{1}{2},-\frac{1}{2}\right)$ (B) $\left(\frac{1}{2},\frac{1}{8}\right)$ (C) $\left(1,-\frac{1}{4}\right)$ (D) $\left(1,\frac{1}{2}\right)$ (E) $(2,2)$

14. If the function f is continuous for all real numbers and if $f(x)=\frac{x^2-4}{x+2}$ when $x\neq-2$, then $f(-2)=$

A

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

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15. The slope of the line normal to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

B

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

Free Response – No Calculators Please

1. The position function of a particle is given by $x(t) = t^3 - 2t^2 - 4t + 6$ for $t \geq 0$.
   a) Find the velocity function
   b) Find the acceleration function
   c) For what value(s) of t, $0 \leq t \leq 4$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0, 4]? Show all work that leads to your conclusion.
   d) Find the total distance traveled by the particle from t = 0 until t = 4. Show all work that leads to your conclusion.

2. Let $f(x) = \sqrt{1 - \sin x}$.
   a) What is the domain of f?
   b) Find f'(x)
   c) What is the domain of f'?
   d) Write an equation for the line tangent to the graph of f at x = 0

1. What is $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{3}+h\right) - \cos\left(\frac{\pi}{3}\right)}{h}$?

a) 0 b) $-\frac{1}{2}$ c) $\frac{1}{2}$

d) $\frac{\sqrt{3}}{2}$ e) $-\frac{\sqrt{3}}{2}$

2. $\lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$ is

a) 1 b) 0 c) e d) e^x

e) undefined

3. The functions f and g are differentiable and have the values shown in the table.

If $A = \left(\frac{f}{g}\right)$ then $A'(2) =$

a) $\frac{23}{25}$ b) $-\frac{23}{4}$

c) $\frac{23}{4}$ d) -7

e) $-\frac{23}{25}$

x	f	f'	g	g'
0	5	1	-7	$\frac{1}{4}$
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

4. The functions f and g are differentiable and have the values shown in the table.

If $A = \sqrt{g(x)}$ then $A'(-2) =$

a) $\frac{9}{8}$

b) impossible

c) $\frac{3}{2}$

d) $\frac{4}{9}$

e) 6

x	f	f'	g	g'
-8	4	3	-2	6
-6	10	12	0	9
-2	16	9	36	18
2	30	15	52	24

5. If $f(4) = 7$ and $f'(4) = 5$, then $f(4.097)$ is approximately __________.

a) 7.902 b) 7.749 c) 7.485

d) 6.932 e) 6.851

6. The position of an object is given by $s = t^2 - 3t + 8$. What is its average velocity for $3 \leq t \leq 5$?

a) 4 b) 3.333 c) 5

d) -5 e) 0.2

7. Given the position function $s = t^3 - 2t + 5$, what is the instantaneous rate of change at $t = 3$?

a) $3t^2 - 2$ b) $3t^2$ c) 27

d) 25 e) 30

8. If $f(x) = \sin(2x)\cos x$, then $f'\left(\frac{\pi}{3}\right) =$

a) $\sqrt{3} + 1$ b) $\frac{5}{4}$ c) $\frac{\pi^2}{3} - 1$

d) $-\frac{5}{4}$ e) $\frac{\pi}{3}$

9. Differentiate: $\frac{1 + \sin x}{1 - \sin x}$

a) -1 b) $-2\sec x$ c) $2\sec x$

d) $\frac{-2}{(1-\sin x)^2}$ e) $\frac{2\cos x}{(1-\sin x)^2}$

10. If $y = \ln\sqrt{\frac{1-x}{1+x}}$, then $\frac{dy}{dx} =$

a) $\frac{1}{1-x^2}$ b) $\frac{1}{1+x^2}$ c) $\frac{-1}{1+x^2}$

d) $\frac{-1}{1-x^2}$ e) 0

11. Assume $f(7) = 0$, $f'(7) = 14$, $g(7) = 1$, and $g'(7) = \frac{1}{7}$. Find $h'(7)$ given $h(x) = \frac{f(x)}{g(x)}$.

a) -14 b) -2 c) 14 d) $\frac{49}{2}$ e) 98

12. Find the derivative of $9x^2 f(x)$.

a) $9x^2 f'(x)$ b) $9x\,[xf(x) + 2f'(x)]$

c) $18xf'(x)$ d) $9x\,[xf'(x) + 2f(x)]$

e) $3x^3 + [f'(x)]^2$

13. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x-7}$ at the point where $x = 16$.

a) $x - 6y = -2$ b) $6x - y = 2$

c) $x + 6y = 2$ d) $x - 6y = 2$

e) $6x + y = -2$

14. The graph of $f(x) = \frac{-5x^2}{7+x^2}$ has a horizontal tangent at $y =$

a) -5 b) 5 c) $\sqrt{7}$ d) $-\sqrt{7}$ e) 0

15. If $f(x) = x^2 e^x$ find a point where the tangent is horizontal.

a) $(0,1)$ b) $(0, e^2)$ c) $(-2, \frac{4}{e^2})$

d) $(0,-2)$ e) $(-2, 4e^2)$

16. If $f(x) = (x-5)^{2/3} + 1$, then the x-value of a vertical tangent is

a) -5 b) 0 c) 1 d) 5 e) $\sqrt[3]{5}$

17. Given a function is defined by $f(x) = \sqrt{x+4}$, for what value(s) of x does the function have one or more vertical tangents?

a) 0 only b) 4 only c) -4 only

d) 0 and 4 e) 0 and -4

18. The points on the graph $y - 3 = \sqrt{16 - 9x^2}$ where vertical tangents exist are

a) $(0,7)$ and $(0,-7)$

b) $(-\frac{4}{3}, 3)$ and $(\frac{4}{3}, 3)$

c) $(-\frac{3}{4}, \frac{1}{3})$ and $(\frac{3}{4}, \frac{1}{3})$

d) $(\frac{16}{3}, 3)$ and $(-\frac{16}{9}, 3)$

e) $(-\frac{4}{3}, -3)$ and $(\frac{4}{3}, -3)$

19. If $y = 8\sin 2x \cos 2x$, then $\frac{d^2y}{dx^2} =$

a) $-128 \sin 2x \cos 2x$ b) $128 \sin 6x$

c) $64 \sin 4x$ d) $32 \sin 2x \cos 2x$

e) $8 \sin 2x \cos 4x$

20. Find the derivative of $y = \sqrt[3]{x^2 + x}$.

a) $\frac{1}{3}(x^2+x)^{-2/3}(2x+1)$

b) $\frac{2}{3}(x^2+x)^{-2/3}(2x-1)$

c) $\frac{3}{2}(x^2+x)^{2/3}(2x+1)$

d) $\frac{x}{3}(x+1)^{-2/3}(2x+1)$

e) $\frac{1}{3}(x^2+x)^{2/3}(2x+1)$

21. Find $\frac{dy}{dx}$ for $y = x^3\sqrt{2x+1}$

a) $\frac{x^2(7x+3)}{\sqrt{2x+1}}$ b) $\frac{3x^2}{2\sqrt{2x+1}}$ c) $\frac{8x^3+3x^2}{2\sqrt{2x^4+x^3}}$

d) $\frac{8x+3}{\sqrt{2x+1}}$ e) $\frac{6x^3+3}{\sqrt{2x+1}}$

22. Find the derivative: $s(t) = \sec\sqrt{t}$

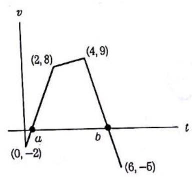
a) $\tan^2\sqrt{t}$ b) $\frac{\sec\sqrt{t}\cdot\tan\sqrt{t}}{2\sqrt{t}}$

c) $\sec\frac{1}{2\sqrt{t}}\cdot\tan\frac{1}{2\sqrt{t}}$ d) $\sec\sqrt{t}\cdot\tan\sqrt{t}$

e) $\frac{\csc\sqrt{t}}{\sqrt{t}}$

23. The graph shows the velocity of an object that is moving along a straight line for t on $[0,6]$.

At what time(s) t does the object reverse direction?

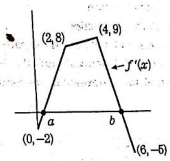


a) 2 and 4 b) a and b c) 4 only

d) 5 only e) a only

24. The graph shows the velocity of an object that is moving along a straight line for t on $[0,6]$.

When is the speed of the object the least?



a) at $t = 0$ b) at $t = 6$

c) at $t = a$ and $t = b$ d) at $t = 2$

e) at $t = -5$