Problem Set 2:

1) $y = 2Kx^2 - x$ $\qquad$ $y = x^3 + cx$

D $\quad y' = 4Kx - 1$ $\qquad$ $y' = 3x^2 + c$

$4Kx - 1 = 3x^2 + c$ at $x = 3$

$12K - 1 = 27 + c$

If diff $\Rightarrow$ cont.

$2Kx^2 - x = x^3 + cx$ at $x = 3$

$18K - 3 = 27 + 3c$

$$-3(12K - c = 28)$$
$$18K - 3c = 30$$
$$\underline{-36K + 3c = -84}$$
$$-18K = -54$$
$$K = 3$$

$12(3) - 1 = 27 + c$

$35 = 27 + c$

$c = 8$

$K + c = 11$

2) $h'(x) = \dfrac{f(x)\, g'(2x) \cdot 2 - g(2x) \cdot f'(x)}{(f(x))^2}$

C $\quad h'(1) = \dfrac{f(1) \cdot g'(2) \cdot 2 - g(2) \cdot f'(1)}{(f(1))^2}$

$h'(1) = \dfrac{4 \cdot -1 \cdot 2 - (-1 \cdot 1)}{4^2} = \dfrac{-8+1}{16} = \dfrac{-7}{16}$

3) $h(x) = f(x) \cdot g(x)$

$h'(x) = f(x) \cdot g'(x) + \cancel{g(x) \cdot \overset{0}{f'(x)}}$ $\qquad$ $f'(x) = 0$

E $\quad$ If $f'(x) = 0$, then $f(x)$ must be a constant.

4) negative slope $\rightarrow f'(x) < 0$

E concave up $\rightarrow f''(x) > 0$

5) $y = 8x^8$

B $y' = 64x^7$

$y'\left(\frac{1}{2}\right) = \frac{1}{2}$

6) $y = \ln(x^2)$

B $y' = \frac{1}{x^2} \cdot 2x$

$y'(e^2) = \frac{1}{(e^2)^2} \cdot 2e^2 = \frac{2}{e^2}$

7) $y = \frac{x-1}{x+1}$ $y' = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

D $y'(1) = \frac{2-0}{4} = \frac{1}{2}$

8) $y = (\cos(3x))^2$

$y' = 2(\cos(3x)) \cdot -\sin(3x) \cdot 3$

A $y' = -6 \sin 3x \cos 3x$

9) $s(t) = t^2 + 4t + 4$

B $\quad v(t) = 2t + 4$

$a(t) = 2$

$a(4) = 2$

10) $h(x) = f(g(x))$

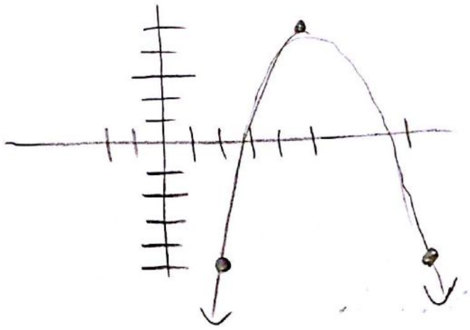
A $\quad h'(x) = f'(g(x)) \cdot g'(x)$

product $h''(x) = f'(g(x)) \cdot g''(x) + g'(x) \cdot f''(g(x)) \cdot g'(x)$

$h''(x) = f''(g(x)) \cdot (g'(x))^2 + f'(g(x)) \cdot g''(x)$

11)

E



I. yes, by IVT

II. yes, there is a max

III. yes, by IVT

12) $y = \cos(2x)$ at $x = \frac{\pi}{4}$

$y' = -\sin(2x) \cdot 2 \qquad y\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$

E $\quad y' = -2\sin(2x)$

$y'\left(\frac{\pi}{4}\right) = -2\sin\left(\frac{\pi}{2}\right) = -2 \cdot 1 = -2$

$y - 0 = -2\left(x - \frac{\pi}{4}\right)$

13) $y = \frac{1}{2}x^2$ $\parallel$ to $2x - 4y = 3$

B

$y' = x$

$\frac{-4y}{-4} = \frac{-2x+3}{-4}$

$y = \frac{1}{2}x - \frac{3}{4}$

$y' = x = \frac{1}{2}$

$x = \frac{1}{2}$

$y\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

$\left(\frac{1}{2}, \frac{1}{8}\right)$

14) $f(x) = \frac{x^2-4}{x+2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}$

A

$f(-2) = -4$

15) $y = 2\ln(\sec x)$

B

$y' = 2 \cdot \frac{1}{\sec x} \cdot \sec x \tan x$

$y'\left(\frac{\pi}{4}\right) = \frac{2}{\sec\frac{\pi}{4}} \cdot \sec\frac{\pi}{4} \tan\frac{\pi}{4}$

$\frac{2}{\frac{2}{\sqrt{2}}} \cdot \frac{2}{\sqrt{2}} \cdot 1$

$\sqrt{2} \cdot \frac{2}{\sqrt{2}} \cdot 1 = 2$ tan line slope

normal slope $= -\frac{1}{2}$

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Derivatives Review :

1) $f(x) = \cos x$

E $f'(x) = -\sin x$

$f'(\frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

2) $f(x) = e^x$

D $f'(x) = e^x$

3) $A = \frac{f}{g}$

$A' = \frac{g \cdot f' - f \cdot g'}{g^2}$

E

$A'(2) = \frac{-5 \cdot 3 - 8 \cdot 1}{(-5)^2}$

$A'(2) = \frac{-23}{25}$

4) $A = (g(x))^{1/2}$

$A' = \frac{1}{2}(g(x))^{-1/2} \cdot g'(x)$

C $A'(-2) = \frac{1}{2}(g(-2))^{-1/2} \cdot g'(-2)$

$= \frac{1}{2}(36)^{-1/2} \cdot 18$

$= \frac{18}{2\sqrt{36}} = \frac{18}{12} = \frac{3}{2}$

5) $y - 7 = 5(x-4)$

C $y - 7 = 5(4.097 - 4)$

$y = 7.485$

6) $s = t^2 - 3t + 8$

C $\frac{s(5) - s(3)}{5-3} = \frac{18-8}{2} = 5$

7) $s = t^3 - 2t + 5$

D $s' = 3t^2 - 2$

$s'(3) = 3(3)^2 - 2$

$= 25$

8) $f(x) = \sin(2x)\cos x$

D $f'(x) = \sin(2x) \cdot (-\sin x) + \cos x(\cos(2x) \cdot 2)$

$f'(\frac{\pi}{3}) = (\sin\frac{2\pi}{3})(-\sin\frac{2\pi}{3}) + (\cos\frac{\pi}{3})(\cos(\frac{2\pi}{3}) \cdot 2)$

$(\frac{\sqrt{3}}{2})(-\frac{\sqrt{3}}{2}) +$

$(\frac{1}{2})(-\frac{1}{2} \cdot 2)$

$-\frac{3}{4} + -\frac{1}{2}$

$= -\frac{5}{4}$

E 9) $y = \frac{1+\sin x}{1-\sin x}$

$y' = \frac{(1-\sin x)\cos x - (1+\sin x)(-\cos x)}{(1-\sin x)^2}$

$y' = \frac{\cos x - \cancel{\sin x\cos x} + \cos x \cancel{+ \sin x\cos x}}{(1-\sin x)^2}$

10) $y = \ln\left(\frac{1-x}{1+x}\right)^{1/2}$

D $\quad y' = \dfrac{1}{\left(\frac{1-x}{1+x}\right)^{1/2}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \left(\dfrac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right)$

$\downarrow$

$\dfrac{-1\cancel{-x}-1+\cancel{x}}{(1+x)^2}$

$y' = \dfrac{1}{2\left(\frac{1-x}{1+x}\right)^{1/2}}\left(\dfrac{-2}{(1+x)^2}\right)$

$y' = \dfrac{\frac{-1}{(1+x)^2}}{\frac{(1-x)^{1/2}}{(1+x)^{1/2}}} = \dfrac{-1}{(1+x)^2} \cdot \dfrac{(1+x)^{1/2}}{(1-x)^{1/2}} =$

$\dfrac{-1}{(1+x)^{1/2}(1-x)^{1/2}}$

$= \dfrac{-1}{\sqrt{(1+x)(1-x)}}$

$= \dfrac{-1}{1-x^2}$

11) $h(x) = \dfrac{f(x)}{g(x)}$

C $\quad h'(x) = \dfrac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$

$h'(7) = \dfrac{1\cdot 14 - 0\cdot \frac{1}{7}}{1^2} = \dfrac{14}{1} = 14$

12) $y = 9x^2 \cdot f(x)$

$y' = 9x^2\cdot f'(x) + f(x)\cdot 18x$

D $\quad y' = 9x(x\cdot f'(x) + 2f(x))$

13) $f(x) = (x-7)^{1/2}$ $\qquad f(16) = 3$

$f'(x) = \frac{1}{2}(x-7)^{-1/2}$

$f'(16) = \frac{1}{2}(9)^{-1/2} = \frac{1}{6}$

A $\qquad y - 3 = \frac{1}{6}(x - 16)$

$6\left(y - 3 = \frac{1}{6}x - \frac{8}{3}\right)$

$6y - 18 = x - 16$

$x - 6y = -2$

14) $f(x) = \dfrac{-5x^2}{7+x^2}$

$f'(x) = \dfrac{(7+x^2)(-10x) - (-5x^2)(2x)}{(7+x^2)^2}$

E

$-70x - \cancel{10x^3} + \cancel{10x^3} = 0$

$-70x = 0$

$x = 0$

$y = 0$

15) $f(x) = x^2 e^x$

$f'(x) = x^2 \cdot e^x + e^x \cdot 2x = 0$

C $\qquad e^x(x^2 + 2x) = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, -2$

$f(0) = 0$

$f(-2) = 4 \cdot e^{-2}$

$\left(-2, \frac{4}{e^2}\right)$

16) $f(x) = (x-5)^{2/3} + 1$

D $f'(x) = \frac{2}{3}(x-5)^{-1/3} = \dfrac{2}{3(x-5)^{1/3}}$

$x = 5$

17) $f(x) = (x+4)^{1/2}$

C $f'(x) = \frac{1}{2}(x+4)^{-1/2} = \dfrac{1}{2\sqrt{x+4}}$

$x = -4$

18) $y - 3 = \sqrt{16-9x^2}$

$y = \sqrt{16-9x^2} + 3 = (16-9x^2)^{1/2} + 3$

B

$y' = \frac{1}{2}(16-9x^2)^{-1/2} \cdot -18x$

$\dfrac{-9x}{\sqrt{16-9x^2}}$

$16 - 9x^2 = 0$

$-9x^2 = -16$

$x^2 = \frac{16}{9}$

$x = \pm \frac{4}{3}$

$f\left(\frac{4}{3}\right) \to y - 3 = \sqrt{16 - 9\left(\frac{4}{3}\right)^2}$

$y = 3$

$\left(\frac{-4}{3}, 3\right), \left(\frac{4}{3}, 3\right)$

19) $y = 8(\sin 2x \cos 2x)$

A $y' = 8\left(\sin 2x \cdot -\sin(2x) \cdot 2 + \cos 2x \cdot \cos(2x) \cdot 2\right)$

$y' = -16(\sin 2x)^2 + 16(\cos 2x)^2$

$y'' = -32(\sin 2x) \cdot \cos 2x \cdot 2 + 32(\cos 2x) \cdot -\sin 2x \cdot 2$

$y'' = -64 \sin 2x \cos 2x - 64 \sin 2x \cos 2x$

$= -128 \sin 2x \cos 2x$

20) $y = (x^2 + x)^{1/3}$

A $y' = \frac{1}{3}(x^2+x)^{-2/3} \cdot (2x+1)$

21) $y = x^3 \cdot (2x+1)^{1/2}$

A $y' = x^3 \cdot \left(\cancel{\tfrac{1}{2}}(2x+1)^{-1/2} \cdot \cancel{2}\right) + (2x+1)^{1/2} \cdot 3x^2$

$\frac{x^3}{\sqrt{2x+1}} + 3x^2\sqrt{2x+1} \cdot \left(\frac{\sqrt{2x+1}}{\sqrt{2x+1}}\right)$

$\frac{x^3 + 3x^2(2x+1)}{\sqrt{2x+1}}$

$\frac{x^3 + 6x^3 + 3x^2}{\sqrt{2x+1}}$ $\quad 7x^3 + 3x^2$

$\frac{x^2(7x+3)}{\sqrt{2x+1}}$

22) $s(t) = \sec(t)^{1/2}$

B $s'(t) = \left(\sec(t^{1/2})\tan(t^{1/2})\right)\left(\frac{1}{2}t^{-1/2}\right)$

23) b) At a and b, the sign of $v(t)$ changes

B

24) at a and b, the velocity is 0

C