Trick or Treat KEY:

Initial Value 1:

① $y' = 2 + x^{-2}$ $\quad y(1) = 6$, $y(3) = ?$

$$\int 2 + x^{-2}dx = 2x - \frac{1}{x} + C$$
$$6 = 2x - \frac{1}{x} + C$$
$$6 = 2 - 1 + C$$
$$C = 5$$
$$f(x) = 2x - \frac{1}{x} + 5$$
$$f(3) = 2(3) - \frac{1}{3} + 5 = \frac{32}{3}$$

$$F(1) + \int_1^3 (2 + x^{-2})dx = F(3)$$
$$6 + \left(2x - \frac{1}{x}\Big|_1^3\right) = F(3)$$
$$6 + \left(6 - \frac{1}{3}\right) - (2 - 1) = F(3)$$
$$F(3) = \frac{32}{3}$$

③ $$F(1) = F(0) + \int_0^1 \cos x^3 dx$$
$$F(1) = 2 + \int_0^1 \cos x^3 dx$$
$$F(1) = 2.932$$

④ $$\int_2^5 e^{-x^2}dx = F(5) - F(2)$$
$$\int_2^5 e^{-x^2}dx = 1 - F(2)$$
$$F(2) = .996$$

Initial Value 2 :

(5) $F(7) = F(6) + \int_6^7 5\sin(t^2)\,dt$

$F(7) = 4 + \int_6^7 5\sin(t^2)\,dt$

$F(7) = 3.837$

(6) $F(3) = F(0) + \int_0^3 2^t\,dt$

$F(3) = 4 + \int_0^3 2^t\,dt$

Total Increase:

$\int_0^3 2^t\,dt = 10.099$ mil

$\downarrow$

Population at $t=3$ hours : 14.099 mil

(7) $s(3) = s(0) + \int_0^3 \frac{t}{1+t^2}\,dt$

$s(3) = 5 + \int_0^3 \frac{t}{1+t^2}\,dt$

$s(3) = 6.151$

Initial Value 3:

8) $3 + 6 \cdot 2 = 9.2$

9) $\int_{-4}^{4} f(x)dx = F(4) - F(-4)$

$\frac{1}{2}\pi(4)^2 = 7 - F(-4)$

$8\pi = 7 - F(-4)$

$8\pi - 7 = -F(-4)$

$F(-4) = 7 - 8\pi$

10) a) $\frac{1}{2}(3)(3) + 5 = \frac{19}{2}$

b) $f(4) = \frac{1}{2}(3)(3) + 5 + \left(-\frac{1}{2}(3)(2)\right) = 6.5$

c) $5 + \frac{9}{2} + (-3) + \frac{1}{2}(\pi \cdot 2^2) = 6.5 + 2\pi$

Initial Value 4:

11) a) $\int_{2}^{6} f(x)dx = 5$ b) 6.5

14) a) $\int_{0}^{2} F'(x)dx = F(2) - F(0)$

$2 = 3 - F(0)$

$F(0) = 1$

$\int_{2}^{6} F'(x)dx = F(6) - F(2)$

$-7 = F(6) - 3$

$F(6) = -4$

$\int_{6}^{8} F'(x)dx = F(8) - F(6)$

$4 = F(8) - (-4)$

$F(8) = 0$

b) Inc: $(0,2) \cup (6,8)$ b/c derivative is pos
Dec: $(2,6)$ b/c derivative is neg

c) CC↑: $(0,1) \cup (4,7)$ b/c 2nd der. is pos
CC↓: $(2,4) \cup (7,8)$ b/c 2nd der. is neg

FTC Apps FRQ 1:

1 a) $h(3) = 11$, $h'(3) = 2$ $h''(3) = -2$

b) $(4,12]$ h' is neg

c) $x=4$ h' pos to neg

d) $x=12$
min value $= -14.5$

$(0,4)$: $\frac{1}{2}(4)(2+4)=12$
$(4,12)$: $\frac{1}{2}(3)(6) + \frac{1}{2}(5)(1+6) = 26.5$

FTC Apps FRQ2: $12 - 26.5 = -14.5$

2 a) $g(-2) = 3$, $g(2) = -6$, $g(8) = -6$

b) $x = -4$

c) $g'(0) = -3$, $g'(5) = 0$, $g'(8) = 3$

d) $g''(3) = 1$, $g''(6) = 1$

FTC Apps FRQ 3:

(3) a) $x = 8$

b) $(3, 9]$

c) 6

d) 3

e) DNE

f) $(0, 6)$

FTC Apps FRQ 4:

(4) a) $g(4) = 5, \quad g'(4) = 3, \quad g''(4) = 0$

b) $[-2, 0) \cup (8, 10]$ b/c g' neg

c) $(4, 10)$ b/c slope of g' neg

d) $x = 4$ b/c slope of g' changes sign

Motion FRQ 1:

(1) a) $\displaystyle\int_0^3 (t^2 + 3t - 10)\,dt = \frac{t^3}{3} + \frac{3t^2}{2} - 10t \Big|_0^3$

$= \left(9 + \frac{27}{2} - 30\right) - 0$

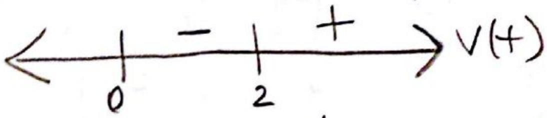
$= -\frac{15}{2}$

b) slowing down

$v(t) = 0 = t^2 + 3t - 10$

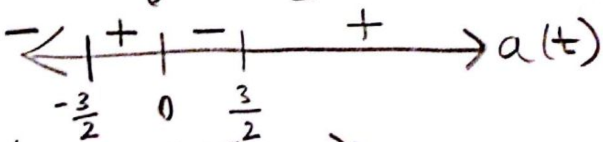
$(t+5)(t-2) = 0$

$t = 2$



$a(t) = 0 = 2t + 3$

$t = -\frac{3}{2}$



slow down: $[0, 2)$

c) $\int_0^5 |t^2+3t-10|\, dt$

Motion FRQ 2:

2 a) $a(t) = -4 \text{ ft/sec}^2$

$v(t) = \int -4\, dt = -4t + c = v(t)$

$12 = -4(0) + c$

$c = 12$

$v(t) = -4t + 12$

$\text{avg } v(t) = \dfrac{\int_0^8 (-4t+12)\,dt}{8} = \dfrac{-2t^2 + 12t \Big|_0^8}{8}$

$= \dfrac{-128+96}{8} = \dfrac{-32}{8} = -4 \text{ ft/sec}$

b) $v(t) = -4t + 12 = -4$

$-4t = -16$

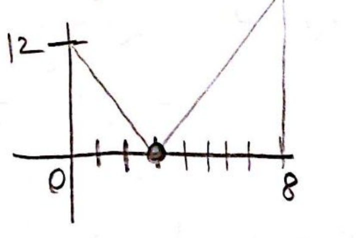
$t = 4 \text{ sec}$

c) v(t) inc when a(t) is pos

$a(t) = -4 \text{ ft/sec}^2$

never

d) $\int_0^8 |-4t+12|\, dt$



$\frac{1}{2}(3)(12) + \frac{1}{2}(5)(20)$

$18 + 50$

$= 68 \text{ ft}$

Motion FRQ 3:

3) a) right when $v(t)$ is pos

$v(t) = 3\cos(2t) = 0$

$t = .785 \quad \pi/4$

$2.356 \quad 3\pi/4$

$3.927 \quad 5\pi/4$

$5.498 \quad 7\pi/4$

right: $(0, .785) \cup (2.356, 3.927) \cup (5.498, 2\pi)$

b) $\int_0^{2\pi} |3\cos(2t)|\,dt = 12$ in

c) $x(0) = 5$, $x(6) = ?$

$x(0) + \int_0^6 v(t) = x(6)$

$5 + \int_0^6 3\cos(2t)\,dt = 4.195$ in

d) speeding up $v(t)$, $a(t)$ same signs

graph on calc:

$(.785, 1.571) \cup (2.356, 3.142) \cup$

$(3.927, 4.712) \cup (5.498, 2\pi)$

Motion FRQ 4:

(4) a) $a(t) = 6t + 6$

$v(t) = \int a(t)\,dt$

$v(t) = 3t^2 + 6t + C$

$-9 = 0 + 0 + C$

$C = -9$

$v(t) = 3t^2 + 6t - 9$

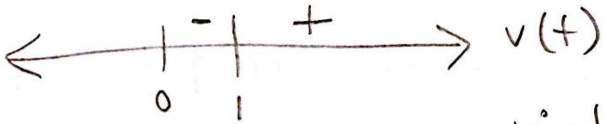
b) right $v(t)$ is pos

$3t^2 + 6t - 9 = 0$

$3(t^2 + 2t - 3) = 0$

$3(t+3)(t-1) = 0$

$t = 1$



right: $(1, \infty)$

c) $x(t) = \int v(t)\,dt$

$x(t) = \int (3t^2 + 6t - 9)\,dt$

$x(t) = t^3 + 3t^2 - 9t + C$

$-27 = 0 + 0 - 0 + C$

$C = -27$

$x(t) = t^3 + 3t^2 - 9t - 27$

Motion MC 1:

(2) absolute value of Area under curve:

B

$\frac{1}{2}(3)(6+2) + \frac{1}{2}(2)(1)$

$12 + 1 = 13$

(5) $a(t) = 2t - 7 \qquad 0 \le t \le 4$

$v(0) = 6$

B

$v(t) = \int (2t - 7)\,dt$

$v(t) = t^2 - 7t + c$

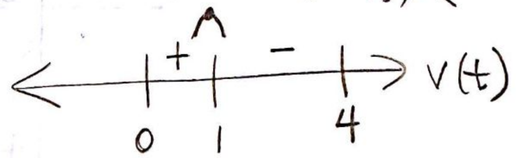
$6 = 0 - 0 + c$

$c = 6$

$v(t) = t^2 - 7t + 6 = 0$

$(t-6)(t-1) = 0$

$t = 1, \cancel{6}$



$x(t) = \int (t^2 - 7t + 6)\,dt$

$x(t) = \frac{t^3}{3} - \frac{7t^2}{2} + 6t$

t	$x(t)$
0	0
1	$17/6$
4	$-32/3$

furthest right at $t = 1$

Motion MC 2:

(8) $a(t) = t + \sin t$

$t = 0,\ v = -2$

B

$v(t) = \int (t + \sin t)\,dt$

$v(t) = \frac{t^2}{2} - \cos t + c$

$-2 = 0 - 1 + c \qquad c = -1$

$v(t) = \frac{t^2}{2} - \cos t - 1 = 0$

$t = 1.48$ sec

(11)

C $\quad v(t) = \cos(2 - t^2)$

$x(0) = 3$

$v(t) = 0 = \cos(2 - t^2)$

$t = .655 \rightarrow A$

$x(A) = 3 + \int_0^A \cos(2 - t^2)\,dt$

$= 2.816$