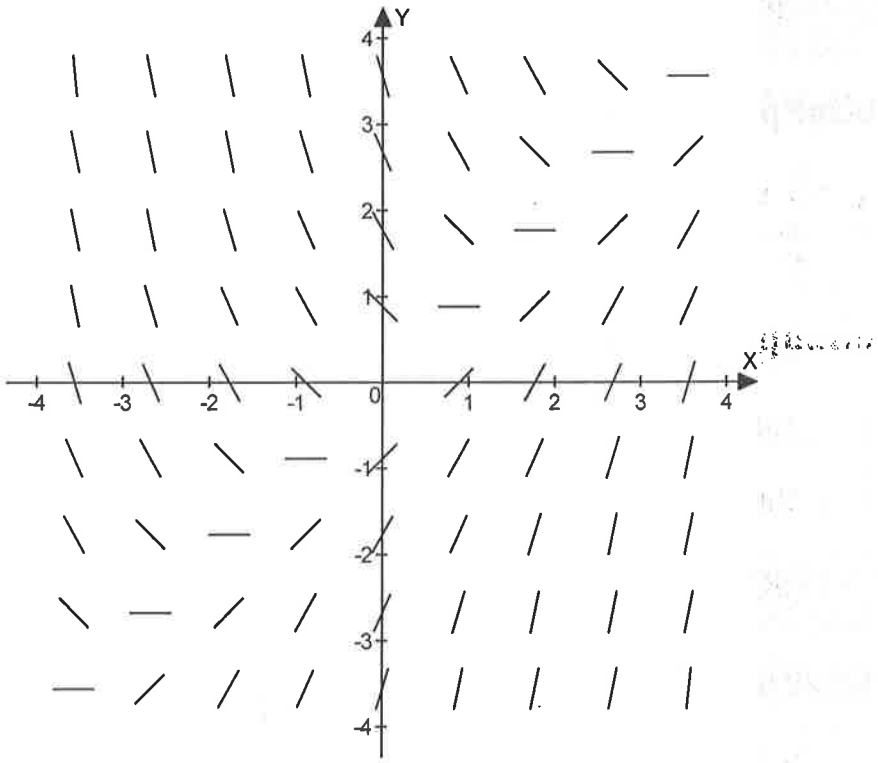
1. Shown below is a slope field for which of the following differential equations?



(A) $\frac{dy}{dx} = y - x$

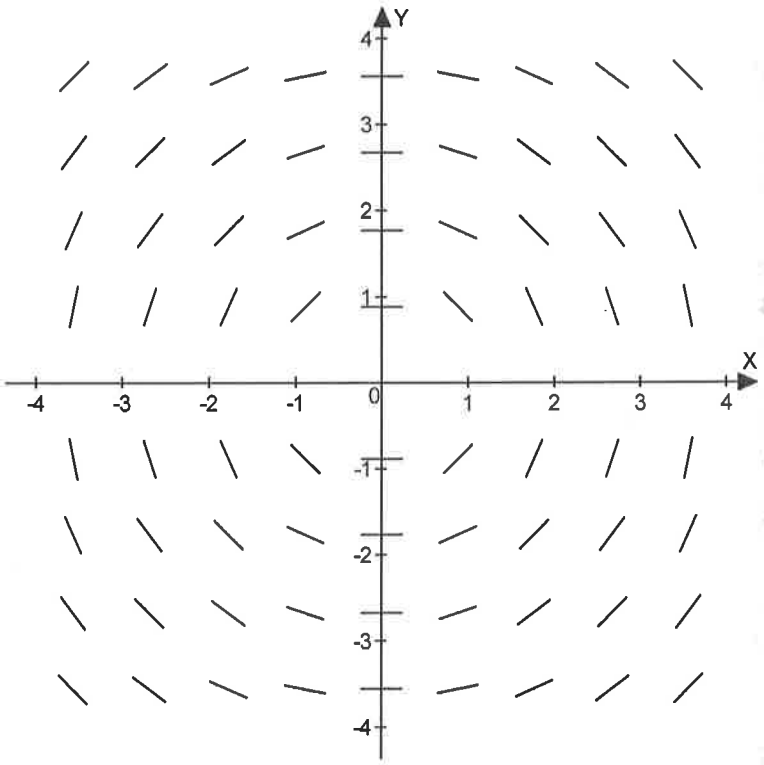
(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = \frac{x}{y}$

(D) $\frac{dy}{dx} = \frac{y}{x}$

(E) $\frac{dy}{dx} = -\frac{x}{y}$

2. Which of the following differential equations could have been used to create the slope field below?



(A) $\frac{dy}{dx} = xy$

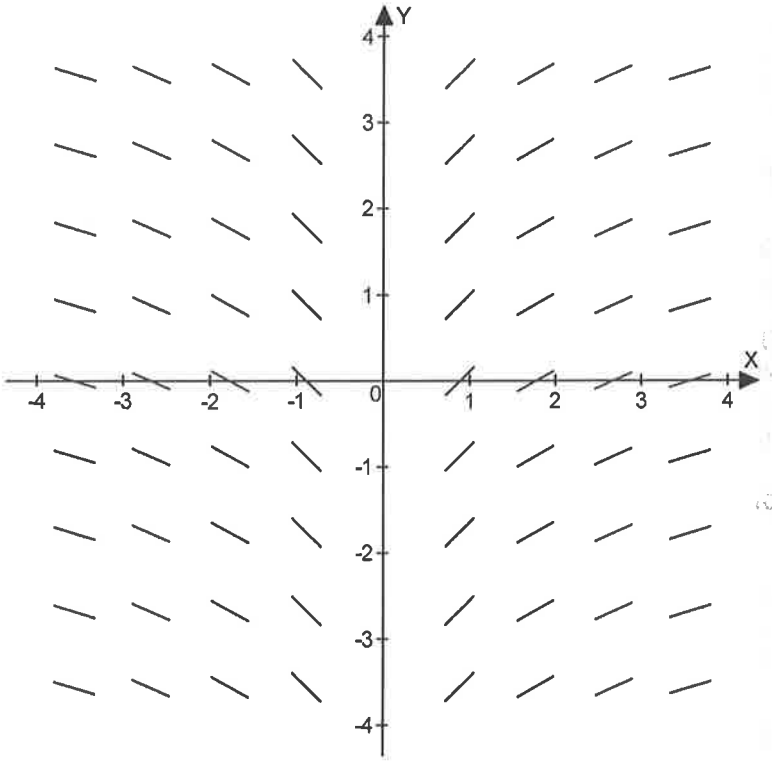
(B) $\frac{dy}{dx} = -xy$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = -\frac{x}{y}$

(E) $\frac{dy}{dx} = y - x$

3. Which of the following could be a solution to the slope field below?



(A) $y = \sin x$

(B) $y = \cos x$

(C) $y = \frac{1}{x}$

(D) $y = -\frac{1}{x}$

(E) $y = \ln|x|$

4. A slope field for the differential equation $\frac{dy}{dx} = 5 - y$ will show

(A) A vertical asymptote at $x = 5$.
(B) A family of parabolas opening upward.
(C) A horizontal asymptote at $y = 5$.
(D) A family of parabolas opening downward.
(E) A line with slope -1 and y-intercept 5.

5. Which of the following could be a solution(s) to the differential equation $\frac{dy}{dt} = 0.2y$?

I. $y = e^{0.2t} + 3$

II. $y = 5e^{0.2t}$

III. $y = 7e^{0.2t}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

6. Which of the following could be a solution(s) to the differential equation $\frac{dy}{dx} = 3x^2$?

I. $y = x^3 + 4$
II. $y = 4x^3$
III. $y = 2x^3 + 1$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

7. If the slope fields of $\frac{dy}{dx} = \frac{1}{x}$ and $\frac{dy}{dx} = g(x)$ are perpendicular, then $g(x)$ could be which of the following?

(A) $g(x) = -\frac{1}{x}$

(B) $g(x) = \frac{1}{x}$

(C) $g(x) = x$

(D) $g(x) = -x$

(E) $g(x) = -\frac{1}{x^2}$

8. Which of the following differential equations would produce a slope field perpendicular to every line in the slope field created by $\frac{dy}{dx} = \sin x$?

(A) $\frac{dy}{dx} = -\csc x$

(B) $\frac{dy}{dx} = \csc x$

(C) $\frac{dy}{dx} = -\sec x$

(D) $\frac{dy}{dx} = \cos x$

(E) $\frac{dy}{dx} = -\cos x$

9. Which of the following is a solution to the differential equation $\frac{dy}{dx}=\frac{y-3}{x}$ $(x\neq 0)$ with $y(1)=1$ and $x>0$?

(A) $y=3-2x$

(B) $y=3+2x$

(C) $y=3+2e^{x-1}$

(D) $y=3-2e^{x-1}$

(E) $y=3+2e^{x}$

10. Which of the following is a solution to the differential equation $\frac{dy}{dx}=x^2\cos^2 y$ with $y(0)=\frac{\pi}{4}$?

(A) $y=\frac{x^3}{3}+1$

(B) $y=\frac{x^3}{3}-1$

(C) $y=\arctan\left(\frac{x^3}{3}+1\right)$

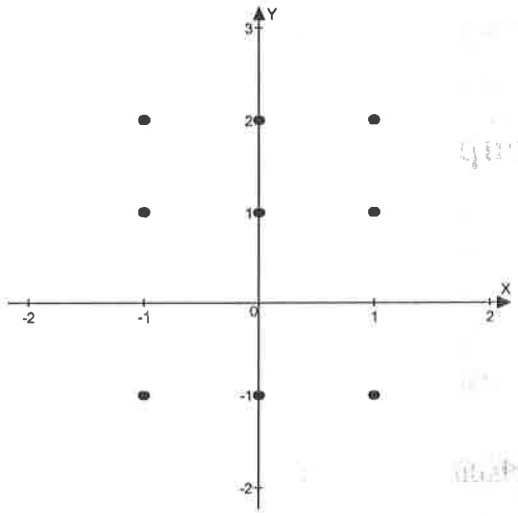
(D) $y=\arctan\left(\frac{x^3}{3}-1\right)$

(E) $y=\tan\left(\frac{x^3}{3}-1\right)$

Free Response 1:

Consider the differential equation $\frac{dy}{dx} = \frac{3x}{y}$ and $y \neq 0$.

(a) On the axes provided, sketch a slope field for the differential equation given at the nine points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

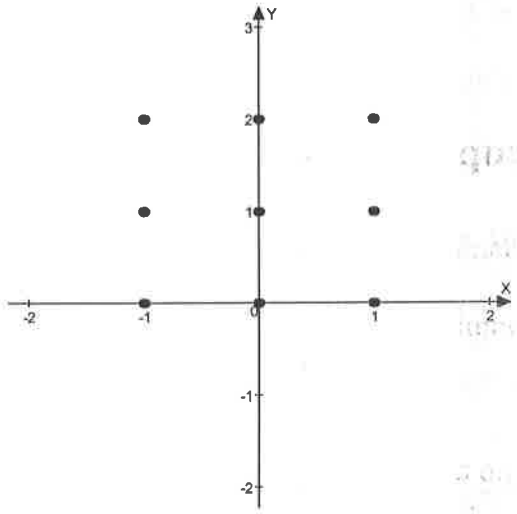
(c) What is the particular solution $y = h(x)$ to the differential equation with the initial condition $h(0) = -1$.

(d) Is the point $h(0) = -1$ a relative minimum, relative maximum, or neither? Justify your answer.

Free Response 2.

Consider the differential equation $\frac{dy}{dx} = x + 2y - 1$.

(a) Sketch a slope field for the given differential equation on the axes provided at the nine points indicated.



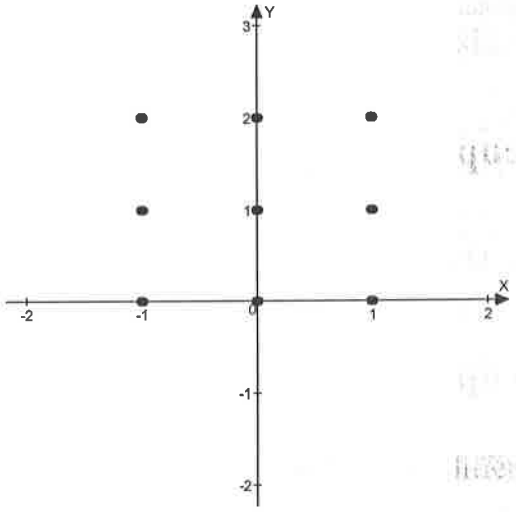
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy – plane in which the solution curves are concave down.

(c) One particular solution to the differential equation passes through the point $(1, 0)$. Is this point a relative minimum, relative maximum, or neither. Justify your answer.

Free Response 3.

Consider the differential equation $\frac{dy}{dx} = y\cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points provided.

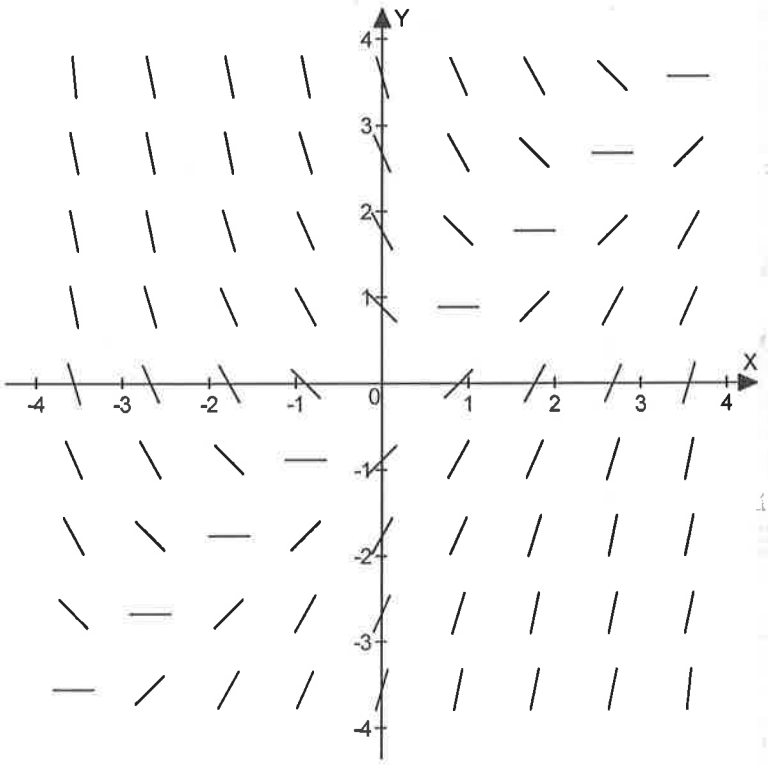


(b) Find the particular solution $y = g(x)$ to the differential equation with initial condition $y(1) = 2$.

(c) Write an equation for the tangent line that passes through the point $(1, 2)$.

Slope Fields and Differential Equations

1. Shown below is a slope field for which of the following differential equations?



(A) $\frac{dy}{dx} = y - x$

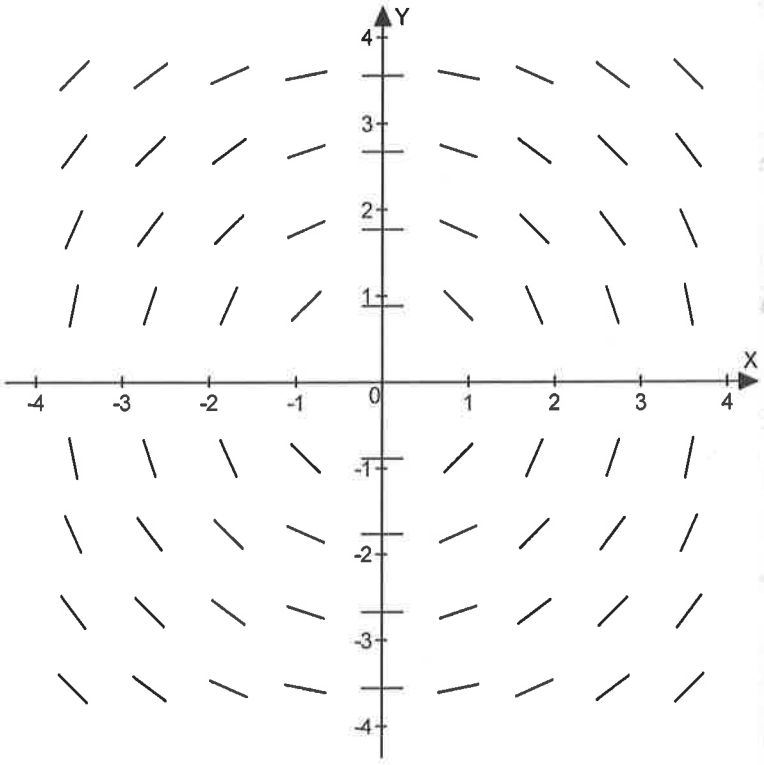
(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = \frac{x}{y}$

(D) $\frac{dy}{dx} = \frac{y}{x}$

(E) $\frac{dy}{dx} = -\frac{x}{y}$

2. Which of the following differential equations could have been used to create the slope field below?



(A) $\frac{dy}{dx} = xy$

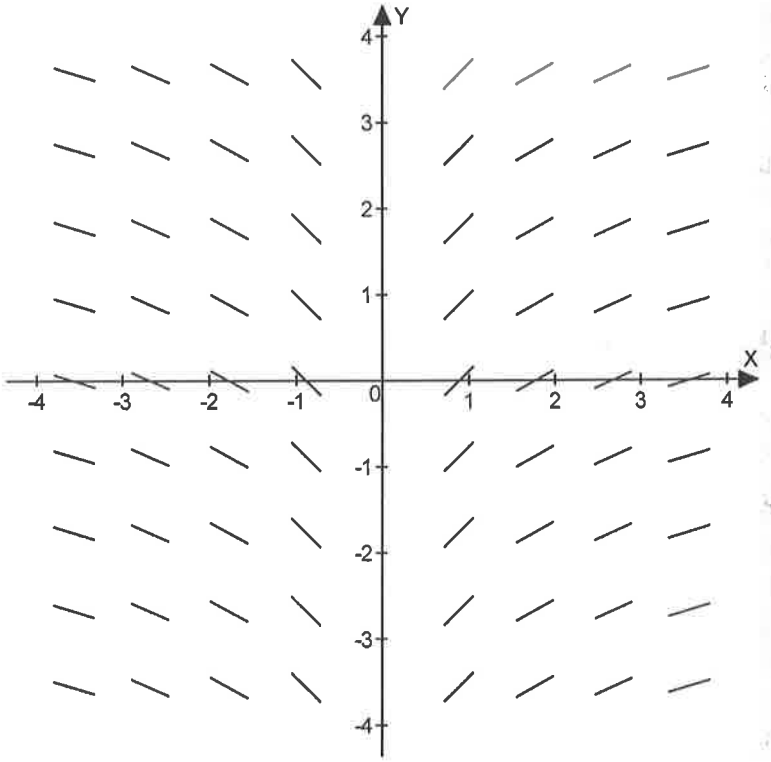
(B) $\frac{dy}{dx} = -xy$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = -\frac{x}{y}$

(E) $\frac{dy}{dx} = y - x$

3. Which of the following could be a solution to the slope field below?



(A) $y = \sin x$

(B) $y = \cos x$

(C) $y = \frac{1}{x}$

(D) $y = -\frac{1}{x}$

(E) $y = \ln|x|$

4. A slope field for the differential equation $\frac{dy}{dx} = 5 - y$ will show

(A) A vertical asymptote at $x = 5$.
(B) A family of parabolas opening upward.
(C) A horizontal asymptote at $y = 5$.
(D) A family of parabolas opening downward.
(E) A line with slope -1 and y-intercept 5.

5. Which of the following could be a solution(s) to the differential equation $\frac{dy}{dt} = 0.2y$?

I. $y = e^{0.2t} + 3$

II. $y = 5e^{0.2t}$

III. $y = 7e^{0.2t}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

6. Which of the following could be a solution(s) to the differential equation $\frac{dy}{dx} = 3x^2$?

I. $y = x^3 + 4$
II. $y = 4x^3$
III. $y = 2x^3 + 1$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

7. If the slope fields of $\frac{dy}{dx} = \frac{1}{x}$ and $\frac{dy}{dx} = g(x)$ are perpendicular, then $g(x)$ could be which of the following?

(A) $g(x) = -\frac{1}{x}$

(B) $g(x) = \frac{1}{x}$

(C) $g(x) = x$

(D) $g(x) = -x$

(E) $g(x) = -\frac{1}{x^2}$

8. Which of the following differential equations would produce a slope field perpendicular to every line in the slope field created by $\frac{dy}{dx} = \sin x$?

(A) $\frac{dy}{dx} = -\csc x$

(B) $\frac{dy}{dx} = \csc x$

(C) $\frac{dy}{dx} = -\sec x$

(D) $\frac{dy}{dx} = \cos x$

(E) $\frac{dy}{dx} = -\cos x$

9. Which of the following is a solution to the differential equation $\frac{dy}{dx}=\frac{y-3}{x}$ $(x\neq 0)$ with $y(1)=1$ and $x>0$?

(A) $y=3-2x$

(B) $y=3+2x$

(C) $y=3+2e^{x-1}$

(D) $y=3-2e^{x-1}$

(E) $y=3+2e^{x}$

10. Which of the following is a solution to the differential equation $\frac{dy}{dx}=x^2\cos^2 y$ with $y(0)=\frac{\pi}{4}$?

(A) $y=\frac{x^3}{3}+1$

(B) $y=\frac{x^3}{3}-1$

(C) $y=\arctan\left(\frac{x^3}{3}+1\right)$

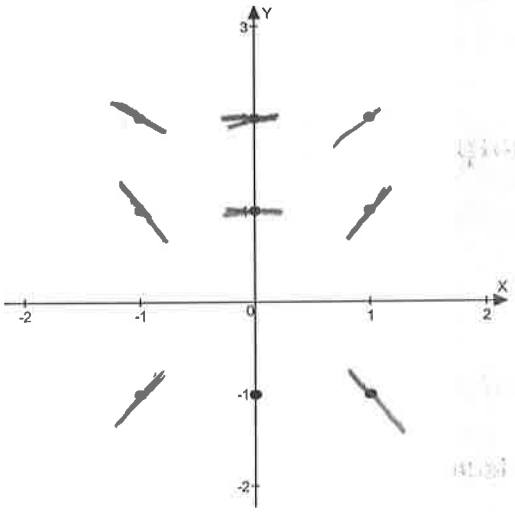
(D) $y=\arctan\left(\frac{x^3}{3}-1\right)$

(E) $y=\tan\left(\frac{x^3}{3}-1\right)$

Free Response 1:

Consider the differential equation $\frac{dy}{dx} = \frac{3x}{y}$ and $y \neq 0$.

(a) On the axes provided, sketch a slope field for the differential equation given at the nine points indicated.



2 pts
1 pt zero slopes
1 pt other slopes

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

$$\frac{3y - \frac{3x}{y}}{y^2}$$

1 pt

(c) What is the particular solution $y = h(x)$ to the differential equation with the initial condition $h(0) = -1$.

$\int y\,dy = \int 3x$

$\frac{1}{2}y^2 = \frac{3}{2}x^2 + C$

$\frac{1}{2} = C$

$\frac{1}{2}y^2 = \frac{3}{2}x^2 + \frac{1}{2}$

$y^2 = 3x^2 + 1$

$y = \pm\sqrt{3x^2+1}$

$y = -\sqrt{3x^2+1}$

4 pts
1 pt sep dy V
1 pt antiderIV
1 pt C
1 pt answer

(d) Is the point $h(0) = -1$ a relative minimum, relative maximum, or neither? Justify your answer.

$h'(0) = 0$

$h''(0) = -3$

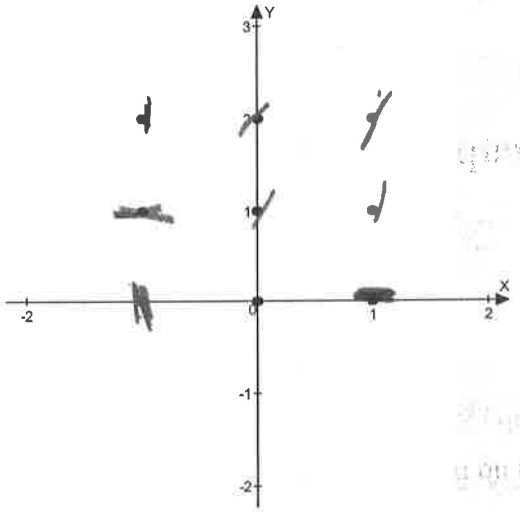
Relative maximum b/c $h'(0) = 0$ and $h''(0)$ is neg.

1 pt answer
1 pt justification
2 pts

Free Response 2.

Consider the differential equation $\frac{dy}{dx} = x + 2y - 1$.

(a) Sketch a slope field for the given differential equation on the axes provided at the nine points indicated.



3pts
1 pt zero slope
1 pt positive slopes
1 pt Negative slopes

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy – plane in which the solution curves are concave down.

3pts
2 pts y''
1 pt description

$\frac{d^2y}{dx^2} = 1 + 2(x + 2y - 1) = 2x + 4y - 1$

Concave down below the line $y = \frac{1}{4} - \frac{1}{2}x$

(c) One particular solution to the differential equation passes through the point (1, 0). Is this point a relative minimum, relative maximum, or neither. Justify your answer.

3pts
1 pt $y' = 0$
1 pt $y'' = 1$
1 pt answer with justification

$\frac{dy}{dx} = 0$

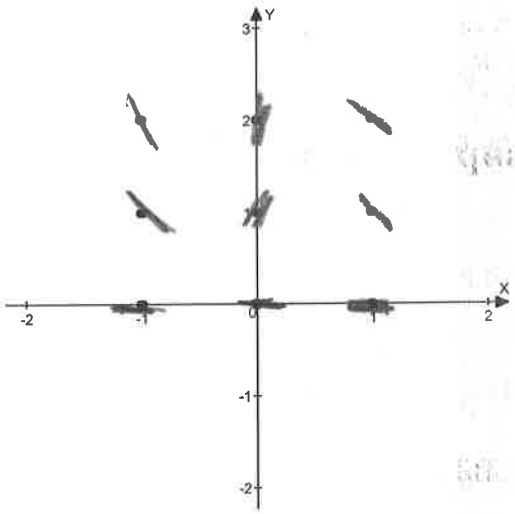
$\frac{d^2y}{dx} = 1$

Relative minimum b/c $y' = 0$ + y'' is positive

Free Response 3.

Consider the differential equation $\frac{dy}{dx} = y\cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points provided.



2 pts
1 pt zero slopes
1 pt other slopes

6 pts
1 pt sep of variables
2 pt antideriv.
1 pt C
1 pt initial cond
1 pt solve for y

(b) Find the particular solution $y = g(x)$ to the differential equation with initial condition $y(1) = 2$.

$\int \frac{dy}{y} = \int \cos \pi x \, dx$

$\ln y = \frac{\sin \pi x}{\pi} + C$

$\ln 2 = C$

$\ln y = \frac{\sin \pi x}{\pi} + \ln 2$

$2e^{\frac{\sin \pi x}{\pi}} = y$

(c) Write an equation for the tangent line that passes through the point (1, 2).

$m = 2\cos \pi$
$= -2$

$y - 2 = -2(x - 1)$

1 pt equation of line