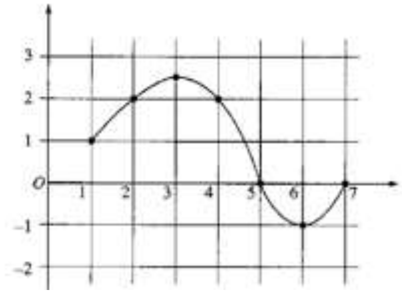
1995 AB6:

The graph of a differentiable function f on the closed interval $[1,7]$ is shown above.



Let $h(x)=\int_1^x f(t)\,dt$ for $1 \le x \le 7$.

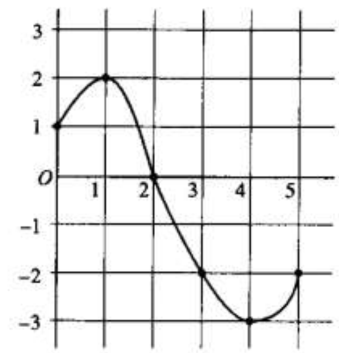
(a) Find $h(1)$.

(b) Find $h'(4)$.

(c) On what interval or intervals is the graph of h concave upward? Justify your answer.

1995 BC6:

Let f be a function whose domain is the closed interval $[0,5]$. The graph of f is shown above.



Graph of f

Let $h(x)=\int_0^{\frac{x}{2}+3} f(t)\,dt$.

(b) Find $h'(2)$.

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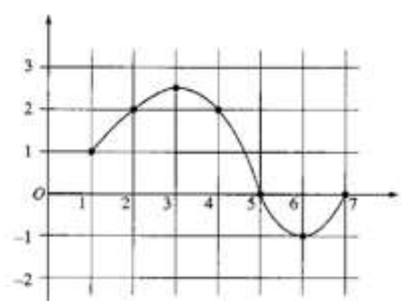

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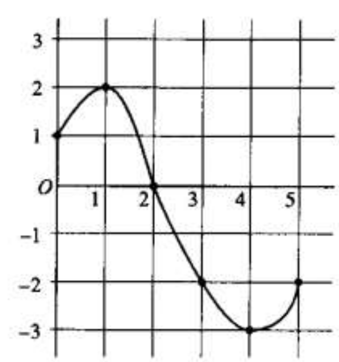

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1995 AB6
Solution

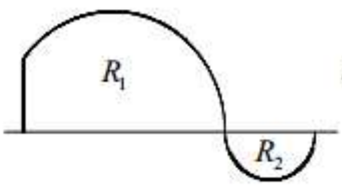
(a) $h(1) = \int_1^1 f(t)\,dt = 0$

(b) $h'(4) = f(4) = 2$

(c) $1 < x < 3$ and $6 < x < 7$
h is concave up when:
- h' is increasing, or
- f is increasing, or
- $h''(x) > 0$

(d) minimum at $x = 1$ because:

h increases on [1,5] and decreases on [5,7], so minimum is at an endpoint



$h(7) = \text{area } R_1 - \text{area } R_2 > 0$ and $h(1) = 0$

(b) $h'(x) = f\left(\frac{x}{2}+3\right)\cdot\frac{1}{2}$
$h'(2) = f(4)\cdot\frac{1}{2} = -\frac{3}{2}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

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(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

$2 : \begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

(b) $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$

Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.

$2 : \begin{cases} 1 : \dfrac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

$2 : \begin{cases} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{cases}$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.

$$\left(g^{-1}\right)'(2) = \frac{1}{g'\left(g^{-1}(2)\right)} = \frac{1}{g'(1)} = \frac{1}{5}$$

An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

$3 : \begin{cases} 1 : g^{-1}(2) \\ 1 : \left(g^{-1}\right)'(2) \\ 1 : \text{tangent line equation} \end{cases}$