Sammit

Test Review for Applications of Derivatives Test

1. What is the x-coordinate of the point of inflection of the graph $y = x^3 + 3x^2 - 45x + 81$

a) -9 b) -5 c) -1 d) 1 e) 3

2. The side of a cube is expanding at a constant rate of 2 cm per second. What is the instantaneous rate of change of the surface area of the cube, in cm^2 per second, when its volume is 27 cubic centimeters?

a) 6 b) 25 c) 36 d) 54 e) 72

3. If $f'(x) = x^3(x+2)^2$, then the graph has inflection points when x =

a) -2 only
b) 0 only
c) -2 and 0
d) -2 and -6/5
e) -2, -6/5 and 0

4. The function $g(x) = \frac{3x^2}{e^{3x}}$ is increasing on which of the following intervals?

a) $(-\infty, 0)$
b) $(-\infty, 2/3)$
c) $(0, 2/3)$
d) $(0, \infty)$
e) $(2/3, \infty)$

5. Suppose $f(x) = x^4 + ax^2$. What is the value of a if f has a local minimum at x = 2

a) -24 b) -8 c) -4 d) -1/2 e) -1/6

6. If $f'(x) = -5(x-3)^2(x-2)$, which of the following features does the graph of f(x) have?

a) A local minimum at x =2 and a local max at x = 3
b) A local max at x = 2 and a local min at x = 3
c) A point of inflection at x = 2 and a local min at x = 3
d) A local min at x = 2 and point of inflection at x = 3
e) A local max at x = 2 and a point of inflection at x = 3

7. If $x + \sin y = \ln y$, then dy/dx =

a) $y + y\cos y$
b) $\frac{y + \cos y - 1}{y}$
c) $\frac{1-y}{y\cos y}$
d) $\frac{y}{y\cos y - 1}$
e) $\frac{y}{1 - y\cos y}$

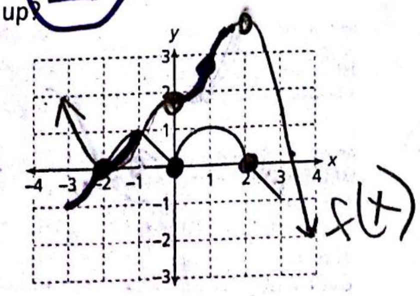
8. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

9. What is the equation of the line normal to the curve $y = e^{2x}\ln(x)$ where x = 1?

(A) $y = e^2(x-1)$ (B) $y = -e^2(x-1)$ (C) $y = -e(x-1)$ (D) $y = -e^{-2}(x-1)$ (E) $y = e^{-2}(x-1)$

10. The graph of f′(x) is given below for $x \in [-3,3]$. On which interval(s) is the function f(x) both increasing and concave up?



(A) (-2, 2) (B) (-2, 0)U(0, 2) (C) (-3, -2) (D) (-2, -1)U(0, 1)

11. What value of c in the open interval (0, 4) satisfies the Mean Value Theorem for $f(x) = \sqrt{3x+4}$?

(A) 0 (B) 3/5 (C) 5/3 (D) 3 (E) 3

ers

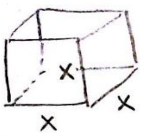
1) C
2) E
3) D
4) C
5) B
6) E
7) E
8) D
9) D
10) d
11) c

(1) $y' = 3x^2 + 6x - 45$

$y'' = 6x + 6 = 0$

$x = -1$

(2)



$\frac{dx}{dt} = 2\text{ cm/s}$

$\frac{dA}{dt} = ?$

when $V = 27\text{ cm}^3$

$V = 27$

$x = 3$

$A = 6x^2$

$\frac{dA}{dt} = 12x\frac{dx}{dt}$

$\frac{dA}{dt} = 12(3)(2)$

$= 72$

(3) $f'(x) = x^3(x+2)^2$

$f''(x) = x^3 \cdot 2(x+2) + (x+2)^2 \cdot 3x^2$

$f''(x) = 2x^3(x+2) + 3x^2(x+2)^2$

$f''(x) = x^2(x+2)[2x + 3(x+2)]$

$f''(x) = x^2(x+2)(5x+6)$

$x = 0, -2, -6/5$

$+ \quad - \quad + \quad +$ $f''(x)$

$-2 \quad -6/5 \quad 0 \quad x$

POI

(4) $g(x) = \frac{3x^2}{e^{3x}}$

$g'(x) = \frac{e^{3x} \cdot 6x - 3x^2 \cdot e^{3x} \cdot 3}{(e^{3x})^2}$

$g'(x) = \frac{6xe^{3x} - 9x^2e^{3x}}{e^{6x}} = 0$

$3xe^{3x}(2 - 3x) = 0$

$3xe^{3x} = 0 \quad 2 - 3x = 0$

$x \cdot e^{3x} = 0 \quad x = \frac{2}{3}$

$x = 0$

$- \quad + \quad -$ $f'(x)$

$0 \quad \frac{2}{3} \quad x$

(5) $f(x) = x^4 + ax^2$

$f'(x) = 4x^3 + 2ax = 0$

$4(2)^3 + 2(a)(2) = 0$

$32 + 4a = 0$

$a = -8$

(6) $f'(x) = -5(x-3)^2(x-2) = 0$

crit pts: $x = 3, 2$

$+ \quad - \quad -$ $f'(x)$

$2 \quad 3 \quad x$

max

$f'(x) = -5(x^2 - 6x + 9)(x-2)$

$-5(x^3 - 2x^2 - 6x^2 + 12x + 9x - 18)$

$f'(x) = -5(x^3 - 8x^2 + 21x - 18)$

$f'(x) = -5x^3 + 40x^2 - 105x + 90$

$f''(x) = -15x^2 + 80x - 105$

$f''(x) = -5(3x^2 - 16x + 21)$

$f''(x) = -5(3x - 7)(x - 3)$

$x = \frac{7}{3}, 3$

$- \quad + \quad -$ $f''(x)$

$7/3 \quad 3 \quad x$

(7) $x + \sin y = \ln y$

$1 + \cos y \frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}\left(\cos y - \frac{1}{y}\right) = -1$

$\frac{dy}{dx} = \left(\frac{-1}{\cos y - \frac{1}{y}}\right)y$

$= \frac{-y}{y\cos y - 1}$

$= \frac{y}{1 - y\cos y}$

⑧ $v(t) = t^3 - 3t^2 + 12t + 4$

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$

$t = 1$

t	$a(t)$
0	12
1	9
3	21

⑨ $y = e^{2x} \cdot \ln x$, $x = 1$ $(1,0)$

normal → $\perp$ to tangent

$y' = e^{2x} \cdot \frac{1}{x} + \ln x \cdot e^{2x} \cdot 2$

$y' = \frac{e^{2x}}{x} + 2e^{2x} \ln x \cdot \frac{x}{x}$

$y' = \frac{e^{2x} + 2xe^{2x} \ln x}{x}$

$\frac{e^2 + 2(1)e^2 \cdot \ln 1}{1}$

$\frac{e^2}{1} + \cancel{2e^2 \cdot 0} = e^2$

slope $\perp$ to e^2 =

$\frac{-1}{e^2} = -e^{-2}$

⑪ $\frac{f(4) - f(0)}{4-0} = \frac{4-2}{4-0} = \frac{2}{4} = \frac{1}{2}$

$f(x) = (3x+4)^{1/2}$

$f'(x) = \frac{1}{2}(3x+4)^{-1/2} \cdot 3$

$\frac{3}{2\sqrt{3x+4}} = \frac{1}{2}$

$6 = 2\sqrt{3x+4}$

$3 = \sqrt{3x+4}$

$9 = 3x + 4$

$5 = 3x$

$x = \frac{5}{3}$