Key

C

A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$.

At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

E

$\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

C

If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

A

$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

E

What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

(A) $(-1, 0)$ (B) $(0, 0)$ (C) $(0, 1)$ (D) $\left(1, \frac{\pi}{4}\right)$ (E) $\left(1, \frac{\pi}{2}\right)$

E

If $F(x) = \int_0^x e^{-t^2} dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$

(D) $e^{-x^2} - 1$ (E) e^{-x^2}

A

An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$

(D) $y = 0$ (E) $\pi x - 2y = 0$

B

At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.

(B) f is decreasing.

(C) f is discontinuous.

(D) f has a relative minimum.

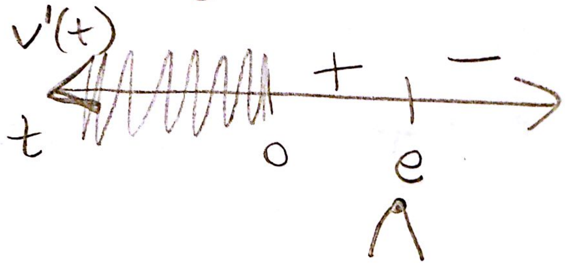
(E) f has a relative maximum.

Test Review #1:

1) $V = \frac{\ln t}{t}$ $V' = \frac{t \cdot \frac{1}{\cancel{t}} - \ln t \cdot 1}{t^2}$

$V' = \frac{1 - \ln t}{t^2} = 0$

$1 - \ln t = 0$
$-\ln t = -1$
$\ln t = 1$
$t = e$



2) $\frac{d}{dx}(\ln e^{2x}) = \frac{d}{dx}(2x \ln e) = \frac{d}{dx}(2x) = 2$

3) $\sin x = e^y$

$\cos x = e^y \cdot \frac{dy}{dx}$

$\cos x = \sin x \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

4) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$ $u = \sin x$
$du = \cos x\,dx$

$u(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$
$u(\frac{\pi}{2}) = 1$

$\int_{\frac{\sqrt{2}}{2}}^{1} \frac{1}{u}\,du = \ln|u| \Big|_{\frac{\sqrt{2}}{2}}^{1} = \cancel{\ln 1} - \ln \frac{\sqrt{2}}{2}$

$= \ln\left(\frac{\sqrt{2}}{2}\right)^{-1}$

$= \ln\left(\frac{2}{\sqrt{2}}\right)$

$\ln \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$= \ln\sqrt{2}$

Scanned by CamScanner

5) $y = (x+1) \cdot \arctan x$

$$y' = (x+1) \cdot \frac{1}{1+x^2} + \arctan x \cdot 1$$

$$y' = \frac{x+1}{x^2+1} + \arctan x$$

$$y'' = \frac{(x^2+1)(1) - \overset{(2x^2+2x)}{(x+1)(2x)}}{(x^2+1)^2} + \frac{1}{1+x^2}\,\frac{(x^2+1)}{(x^2+1)}$$

$$y'' = \frac{\cancel{x^2}+1-\cancel{2x^2}-2x+\cancel{x^2}+1}{(x^2+1)^2}$$

$$y'' = \frac{-2x+2}{(x^2+1)^2} = 0 \qquad -2(x-1)=0$$

$$x = 1$$

y'' + | −

x 1

$$y(1) = (1+1) \cdot \arctan(1) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$$

$$\left(1, \frac{\pi}{2}\right)$$

6) $F(x) = \int_0^x e^{-t^2}$

$$F'(x) = e^{-x^2}$$

Scanned by CamScanner

7) $y = \arcsin\left(\frac{x}{2}\right)$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$$

$$y'(0) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$y(0) = \arcsin\left(\frac{0}{2}\right) = 0$$

$$y - 0 = \frac{1}{2}(x-0)$$

$$y = \frac{1}{2}x$$

$$2y = x$$

$$x - 2y = 0$$

8) $f(x) = x^2 + e^{-2x}$

$$f'(x) = 2x + e^{-2x} \cdot (-2)$$

$$f'(x) = 2x - 2e^{-2x}$$

$$f'(0) = 0 - 2e^0$$

$$f'(0) = 0 - 2 = -2$$

f is dec

'72 BC6 :

a) $\lim_{x \to 0^+} \frac{x}{\ln x} = 0 \neq f(0) = 1$

f is not cont. at $x=0$. f is cont. at all other values of x except $x = 1, -1$ where it is not defined

b) $f(-x) = \frac{-(-x)}{\ln(-(-x))} = \frac{x}{\ln x} = f(x)$
$x > 0$

$x < 0$ $\quad f(-x) = \frac{-x}{\ln(-x)} = f(x)$

(i) symmetric about y-axis since $f(-x) = f(x)$ for all x

(ii) not symmetric about origin since $f(-x) \neq f(x)$

c) and d)

$f'(x) = \frac{\ln x - 1}{(\ln x)^2}$ $\quad x > 0$

$f'(x) = \frac{-\ln(-x)+1}{(\ln(-x))^2}$ $\quad x < 0$

- For $x > 0$, $f'(x) = 0$ at $x = e$ so (e, e) is rel min
- By symmetry, $(-e, e)$ is rel min also

$f''(x) = \frac{\frac{1}{x}(\ln x)^2 - (\ln x - 1)\left(\frac{2}{x}\right)\ln x}{(\ln x)^4}$ $\quad x > 0$

- $f(x) < 0$ for $-1 < x < 0$ and $0 < x < 1$
(0,1) is rel max

Scanned by CamScanner

'79 AB2

a) $f'(x) = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x)$

$f'(x) > 0$ when $1-2x > 0$

inc $0 \leq x < \frac{1}{2}$

$f'(x) < 0$ when $1-2x < 0$

dec $\frac{1}{2} < x \leq 10$

b) $f'(x) = 0 \Rightarrow e^{-2x}(1-2x) = 0$

crit pt $x = \frac{1}{2}$

$0 < x < \frac{1}{2} \Rightarrow f'(x) > 0$

$\frac{1}{2} < x < 10 \Rightarrow f'(x) < 0$

graph of f inc then dec on

$0 \leq x \leq 10$ so abs max is $\left(\frac{1}{2}, \frac{1}{2e}\right)$

Abs min at endpt:

$f(0) = 0 \quad f(10) = \frac{10}{e^{20}}$

Abs min: $(0,0)$

'80 AB4/BC1

a) $a = 10e^{2t}$

$V = \int 10e^{2t}dt = 5e^{2t} + C$

$v(0) = 5 \Rightarrow C = 0$

$V = 5e^{2t}$

b) $v = 5$ when $t = 0$

$v = 15 \Rightarrow 5e^{2t} = 15$

$t = \frac{1}{2}\ln 3$

$\text{Dist} = \int_0^{\frac{1}{2}\ln 3} 5e^{2t}dt = \frac{5}{2}e^{2t}\Big|_0^{\frac{1}{2}\ln 3}$

$= \frac{5}{2}e^{\ln 3} - \frac{5}{2} = 5$

c) $S = \int 5e^{2t}dt = \frac{5}{2}e^{2t} + C$

$s(0) = 0 \Rightarrow C = -\frac{5}{2}$

$S = \frac{5}{2}e^{2t} - \frac{5}{2}$