A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim_{x\to 0} \frac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

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(a) $\lim_{n\to\infty}\left|\dfrac{\dfrac{(n+2)x^{n+1}}{3^{n+2}}}{\dfrac{(n+1)x^n}{3^{n+1}}}\right| = \lim_{n\to\infty}\left|\dfrac{(n+2)}{(n+1)}\dfrac{x}{3}\right| = \left|\dfrac{x}{3}\right| < 1$

At $x = -3$, the series is $\sum_{n=0}^{\infty}(-1)^n\dfrac{n+1}{3}$, which diverges.

At $x = 3$, the series is $\sum_{n=0}^{\infty}\dfrac{n+1}{3}$, which diverges.

Therefore, the interval of convergence is $-3 < x < 3$.

4 :
- 1 : sets up ratio test
- 1 : computes limit
- 1 : conclusion of ratio test
- 1 : endpoint conclusion

(b) $\lim_{x\to 0}\dfrac{f(x)-\frac{1}{3}}{x} = \lim_{x\to 0}\left(\dfrac{2}{3^2}+\dfrac{3}{3^3}x+\dfrac{4}{3^4}x^2+\cdots\right) = \dfrac{2}{9}$

1 : answer

(c) $\int_0^1 f(x)\,dx = \int_0^1\left(\dfrac{1}{3}+\dfrac{2}{3^2}x+\dfrac{3}{3^3}x^2+\cdots+\dfrac{n+1}{3^{n+1}}x^n+\cdots\right)dx$

$= \left(\dfrac{1}{3}x+\dfrac{1}{3^2}x^2+\dfrac{1}{3^3}x^3+\cdots+\dfrac{1}{3^{n+1}}x^{n+1}+\cdots\right)\Big|_{x=0}^{x=1}$

$= \dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+\cdots+\dfrac{1}{3^{n+1}}+\cdots$

3 :
- 1 : antidifferentiation of series
- 1 : first three terms for definite integral series
- 1 : general term

(d) The series representing $\int_0^1 f(x)\,dx$ is a geometric series.

Therefore, $\int_0^1 f(x)\,dx = \dfrac{\frac{1}{3}}{1-\frac{1}{3}} = \dfrac{1}{2}$.

1 : answer