Review 9.1 – 9.6

1) $\lim_{n\to\infty} \frac{n^2-3}{2+2n^2} = \frac{1}{2}$ conv

2) $\lim_{n\to\infty} \frac{\sin(n)}{\ln(n+1)} \to$ alternates $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ conv

3) $\frac{2^n}{3^{n-2}}$ $\frac{2^n}{3^n \cdot 3^{-2}} = \frac{9 \cdot 2^n}{3^n} = 9 \cdot \left(\frac{2}{3}\right)^n$

$\lim_{n\to\infty} 9\left(\frac{2}{3}\right)^n = 0$ conv

4) $\sin\left[\left(\frac{n}{2}\right)\pi\right]$ div

$\hookrightarrow \sin\frac{\pi}{2}, \sin\pi, \sin\frac{3\pi}{2}, \sin 2\pi$

$1, 0, -1, 0$

6) $n^{1/n}$ $\lim_{n\to\infty} n^{1/n}$

$\ln y = \lim_{n\to\infty} \frac{1}{n}\ln n$

$\ln y = \lim_{n\to\infty} \frac{\ln n}{n}$

$\ln y = 0$

$y = e^0 = 1$ conv

7) $\frac{1}{1-\frac{1}{2}} = 2$

8) $r = -\frac{4}{9}$ $\frac{1}{1-\left(-\frac{4}{9}\right)} = \frac{9}{13}$

$a = 1$

9) $r = \frac{e}{\pi}$ $\frac{\pi}{1-\frac{e}{\pi}} = \frac{\pi^2}{\pi - e}$

$a = \pi$

10) $\frac{1}{n+3^n}$ DCT: $\frac{1}{3^n} = \left(\frac{1}{3}\right)^n$ big conv.

$\therefore$ small conv

11) $\frac{(-2)^{2n}}{n^n} = \left(\frac{(-2)^2}{n}\right)^n = \left(\frac{4}{n}\right)^n$ $\sqrt[n]{\left(\frac{4}{n}\right)^n} = \frac{4}{n}$

$\lim_{n\to\infty} \frac{4}{n} = 0$ $\therefore$ conv by root

12) $\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ LCT: $\frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5} \cdot \frac{n^2}{1} = 1$ $\therefore$ conv

13) $(-1)^n \cdot \frac{n}{n+2}$ $\lim_{n\to\infty} \frac{n}{n+2} = 1$ $\therefore$ div

14) $\tan\left(\frac{1}{n}\right)$ DCT: $\frac{1}{n}$ small div

$\therefore$ divergent

$\tan\left(\frac{1}{\frac{4}{\pi}}\right) > \frac{1}{\frac{4}{\pi}}$

$\tan\frac{\pi}{4} > \frac{\pi}{4}$

$1 > \frac{\pi}{4}$

15) $\frac{n^2 \cdot 2^{n-1}}{(-5)^n} = \frac{n^2 \cdot 2^n \cdot 2^{-1}}{(-5)^n} = \frac{n^2 \cdot 2^n}{2 \cdot (-5)^n}$

Ratio: $\lim_{n\to\infty} \frac{(n+1)^2 \cdot 2^{n+1}}{2 \cdot (-5)^{n+1}} \cdot \frac{2 \cdot (-5)^n}{n^2 \cdot 2^n}$

$\lim_{n\to\infty} \frac{(n+1)^2 \cdot \cancel{2^n} \cdot 2}{\cancel{2} \cdot \cancel{(-5)^n}(-5)^1} \cdot \frac{\cancel{2} \cdot \cancel{(-5)^n}}{n^2 \cdot \cancel{2^n}}$

$\lim_{n\to\infty} \frac{2(n+1)^2}{-5n^2} = -\frac{2}{5}$ $\therefore$ conv

$\left|\frac{-2}{5}\right| < 1$

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16) $\left(\frac{\ln k}{k}\right)^k$ $\sqrt[k]{\left(\frac{\ln k}{k}\right)^k} = \frac{\ln k}{k}$ Root

$\lim_{k\to\infty} \frac{\ln k}{k} = 0$ $\therefore$ conv

17) $\frac{n^2+1}{5^n}$ Ratio

$\lim_{n\to\infty} \frac{(n+1)^2+1}{5^{n+1}} \cdot \frac{5^n}{n^2+1}$

$\lim_{n\to\infty} \frac{(n+1)^2+1}{\cancel{5^n}\cdot 5} \cdot \frac{\cancel{5^n}}{n^2+1} = \frac{1}{5}$ $\therefore$ conv

18) $\frac{1}{n\sqrt{\ln n}}$ Integral $\int_2^\infty \frac{1}{x\sqrt{\ln x}}\,dx$ $u = \ln x$
$du = \frac{1}{x}dx$

$\int u^{-1/2}\,du = 2\sqrt{\ln x}\,\Big|_2^b$

$\lim_{b\to\infty} 2\sqrt{\ln x}\,\Big|_2^b$ $\therefore$ diverges

19) $\frac{(2^k)^k}{(k^2)^k}$ Root: $\left(\frac{2^k}{k^2}\right)^k$ $\sqrt[k]{\left(\frac{2^k}{k^2}\right)^k} = \frac{2^k}{k^2}$

$\lim_{k\to\infty} \frac{2^k}{k^2} = \infty$ $\therefore$ div

20) $\frac{(-1)^{n+1}}{n\ln n}$ $\lim_{n\to\infty} \frac{1}{n\ln n} = 0$ ✓

$\frac{1}{(n+1)\ln(n+1)} \le \frac{1}{n\ln n}$

$n\ln n \le (n+1)\ln(n+1)$ ✓

$\therefore$ conv by AST

21) $k^2e^{-k} = \frac{k^2}{e^k}$

Ratio: $\lim_{k\to\infty} \frac{(k+1)^2}{e^{k+1}} \cdot \frac{e^k}{k^2}$ $\quad \frac{(k+1)^2}{\cancel{e^k}\cdot e} \cdot \frac{\cancel{e^k}}{k^2} = \frac{1}{e}$

$\therefore$ conv

22) $\frac{3^n n^2}{n!}$

Ratio: $\lim_{n\to\infty} \frac{3^{n+1}\cdot(n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n\cdot n^2}$

$\frac{\cancel{3^n}\cdot 3(n+1)^2}{\cancel{(n+1)\cdot n!}} \cdot \frac{\cancel{n!}}{\cancel{3^n}\cdot n^2} = 0 \quad \therefore$ conv

23) $\frac{5^k}{3^k+4^k}$ $\quad$ n^{th} term:

$\lim_{n\to\infty} \frac{5^n}{3^n+4^n} = \infty \quad \therefore$ div

24) $(-1)^n \cdot 2^{1/n}$ $\quad \lim_{n\to\infty} 2^{1/n} = 1 \quad \therefore$ div by AST

25) $3\left(\frac{3}{4}\right)^n$ con by geo

26) $2\sqrt[3]{n^{-5}} = \frac{2}{n^{5/3}}$ conv by p-series

27) $\frac{n}{\ln(n+1)}$ $\quad$ n^{th} term: $\lim_{n\to\infty} \frac{n}{\ln(n+1)} = \infty \quad \therefore$ div

28) $\frac{(n+2)!}{2^n \cdot n!}$

Ratio: $\lim_{n\to\infty} \frac{(n+3)!}{2^{n+1}\cdot(n+1)!} \cdot \frac{2^n \cdot n!}{(n+2)!}$

$\lim_{n\to\infty} \frac{(n+3)\cancel{(n+2)!}}{\cancel{2^n}\cdot 2(n+1)\cancel{n!}} \cdot \frac{\cancel{2^n}\cdot \cancel{n!}}{\cancel{(n+2)!}} \to \frac{n+3}{2(n+1)} = \frac{1}{2}$

$\therefore$ conv

29) $\frac{n}{2n+1}$ $\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$ $\therefore$ div by n^{th} term

30) $\frac{1}{n(n+1)}$ $\frac{1}{n^2+n}$ compare to $\frac{1}{n^2}$ big conv

$\therefore \frac{1}{n(n+1)}$ conv by DCT

or Integral Test

31) $\frac{n+1}{n!}$ Ratio: $\lim_{n\to\infty} \frac{(n+2)}{(n+1)!} \cdot \frac{n!}{n+1}$

$\lim_{n\to\infty} \frac{(n+2)}{(n+1)\cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{(n+1)} = 0$ $\therefore$ conv

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32) $\frac{\ln n}{2^n}$ Ratio: $\lim_{n\to\infty} \frac{\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{\ln n}$

$\lim_{n\to\infty} \frac{\ln(n+1)}{\cancel{2^n}\cdot 2} \cdot \frac{\cancel{2^n}}{\ln n} = \frac{1}{2}$

$\therefore$ conv

33) $\frac{e^{\pi/2}}{\pi^n} = e^{\pi/2}\left(\frac{1}{\pi}\right)^n$ $\therefore$ conv by geo

34) $\frac{(-1)^K}{\sqrt{K}}$ $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓ $\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$ conv by AST

$\sqrt{n} \leq \sqrt{n+1}$ ✓

$\sum \frac{1}{K^{1/2}}$ div by p-series $\therefore$ conv. conditionally

35) $\frac{\ln K}{(-3)^K} = \frac{\ln K}{(-1)^K \cdot 3^K}$ $\lim_{n\to\infty} \frac{\ln n}{3^n} = 0$ ✓

$\frac{\ln(n+1)}{3^{n+1}} \leq \frac{\ln n}{3^n}$

$\ln(n+1)\cdot 3^n \leq \ln n \cdot 3^{n+1}$ ✓

Conv by AST

$\sum \frac{1}{3^K} \rightarrow \left(\frac{1}{3}\right)^K$ conv by geo

$\therefore$ conv absolutely

36) $\frac{(-2)^K}{K^2} \rightarrow \frac{(-1)^K \cdot 2^K}{K^2}$ $\lim_{n\to\infty} \frac{2^n}{n^2} = \infty$ $\therefore$ div

37) $\frac{(-1)^K \cdot K^3}{e^{K^4}}$ $\lim_{n\to\infty} \frac{n^3}{e^{n^4}} = 0$ ✓ (or integral)

$\frac{(n+1)^3}{e^{(n+1)^4}} \leq \frac{n^3}{e^{n^4}}$ Conv by AST

$\sum \frac{K^3}{e^{K^4}}$ DCT: $\underbrace{\frac{1}{e^K} = \left(\frac{1}{e}\right)^K}_{\text{big conv}}$ $(n+1)^3 \cdot e^{n^4} \leq n^3\left(e^{(n+1)^4}\right)$ ✓

$\therefore$ ABsolutely conv.

38) $\frac{(-2)^k}{k!} \to \frac{(-1)^k(2)^k}{k!}$

$\lim_{n\to\infty} \frac{2^n}{n!} = 0$ ✓

$\frac{2^{n+1}}{(n+1)!} \leq \frac{2^n}{n!}$

$2^{n+1}(n!) \leq 2^n(n+1)!$ ✓

Conv by AST

$\sum \frac{2^k}{k!}$

Ratio: $\frac{2^{k+1}}{(k+1)!} \cdot \frac{k!}{2^k}$

$\frac{\cancel{2^k}\cdot 2}{(k+1)\cdot \cancel{k!}} \cdot \frac{\cancel{k!}}{\cancel{2^k}} = 0$ ∴ conv by ratio

∴ absolutely conv

39) $\frac{(-1)^k}{k\cdot(\ln k)^2}$

$\lim_{n\to\infty} \frac{1}{n(\ln n)^2} = 0$ ✓

$\frac{1}{(n+1)(\ln(n+1))^2} \leq \frac{1}{n(\ln n)^2}$

$n(\ln n)^2 \leq (n+1)(\ln(n+1))^2$ ✓

Conv by AST

$\sum \frac{1}{n(\ln n)^2}$

Integral: $\int_3^\infty \frac{1}{x(\ln x)^2}dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$\int u^{-2}\,du = \frac{-1}{u}$

$\frac{-1}{\ln x}$

$\lim_{b\to\infty}\int_3^b \frac{1}{x(\ln x)^2}dx = \lim_{b\to\infty} \frac{-1}{\ln x}\Big|_3^b$

$\lim_{b\to\infty}\left(\frac{-1}{\ln b} - \left(\frac{-1}{\ln 3}\right)\right) = \frac{1}{\ln 3}$ ∴ Conv

∴ ABS Conv !