Packet KEY 9.2 – 9.3 (not including p-series)

$\boxed{9.2}$ Ex 1:

A. $\frac{1}{2^{1-1}} = \frac{1}{2^0} = 1$

B. $\frac{1}{2^0} + \frac{1}{2^1} = 1 + \frac{1}{2}$

C. $1 + \frac{1}{2} + \frac{1}{4}$

D. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \frac{1}{2^{n-1}}$

Ex 2:

A. 8

B. $r = \frac{1}{5}$

C. $a_n = 8\left(\frac{1}{5}\right)^{n-1}$

D. $8 + \frac{8}{5} + \frac{8}{25} + \frac{8}{125} = \frac{1248}{125}$

E. $S = \frac{a}{1-r} = \frac{8}{1-\frac{1}{5}} = 10$

F. $\lim_{n\to\infty} 10\left(1-\left(\frac{1}{5}\right)^n\right) = 10$

Ex 3:

A. $\sum_{n=0}^{\infty} 2\left(\frac{1}{3^n}\right) + \sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n} \rightarrow \frac{2^n \cdot 2^{-1}}{5^n} = \frac{1}{2}\left(\frac{2}{5}\right)^n$

$\sum_{n=0}^{\infty} 2 \cdot \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{2}{5}\right)^n$

$\left|\frac{1}{3}\right| < 1$ $\qquad$ $\left|\frac{2}{5}\right| < 1$

$\frac{a}{1-r} = \frac{2}{1-\frac{1}{3}}$ $\qquad$ $\frac{a}{1-r} = \frac{\frac{1}{5}}{1-\frac{2}{5}}$

$3 + \frac{1}{3}$

$\frac{10}{3}$

B. $\sum_{n=0}^{\infty} \frac{4^n - 7^n}{5^n} \longrightarrow \frac{4^n}{5^n} - \frac{7^n}{5^n}$

$$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n - \sum_{n=0}^{\infty} \left(\frac{7}{5}\right)^n$$

$r = \frac{7}{5} > 1$

$\therefore$ entire series diverges

Ex 4:

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$1 = A(n+1) + B(n)$

$n=-1: \; 1 = -1B \quad B = -1$

$n=0: \; 1 = 1A \quad A = 1$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{5}\right) + \ldots$$

$= \boxed{1}$ converges

Ex 5:

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n(n+1)} = 0$ $\therefore$ sequence converges to 0

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$ $\therefore$ series converges to 1 (as shown in Ex 4)

Ex 6: (Don't need to know, but FUN!)

$.050505 = \frac{5}{10^2} + \frac{5}{10^4} + \frac{5}{10^6}$ $\quad a = \frac{5}{10^2} \quad r = \frac{1}{10^2}$

$$\sum_{n=0}^{\infty} \left(\frac{5}{10^2}\right)\left(\frac{1}{10^2}\right)^n = \frac{a}{1-r} = \frac{\frac{5}{10^2}}{1 - \frac{1}{10^2}} = \frac{5}{99}$$

n^{th} Term Test :

Ex 1 :

A. $\lim_{n\to\infty} \frac{n^2}{5n^2+4} = \frac{1}{5} \neq 0$ $\therefore$ diverges

B. $\lim_{n\to\infty} \frac{n}{n^3+1} = 0$ $\therefore$ unable to determine at this time

C. $\lim_{n\to\infty} \frac{1}{n} = 0$ $\therefore$ unable to determine at this time

$\boxed{9.3}$

Ex 1 :

A. $\sum_{n=1}^{\infty} \frac{4^n}{5^{n+1}}$

$\sum_{n=1}^{\infty} \frac{1}{5}\left(\frac{4}{5}\right)^n$

$\left|\frac{4}{5}\right| < 1$

$\therefore$ converges to

$\frac{a}{1-r} = \frac{\frac{4}{25}}{1-\frac{4}{5}}$

$= \frac{4}{5}$

B. $\lim_{n\to\infty} \frac{5n^2+3}{2n^2} = \frac{5}{2} \neq 0$

$\therefore$ diverges

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Exploring the Integral of a Series:
See Proof of Integral Test Video on Weebly

Ex 2:

A. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

f is positive, continuous, + decreasing

$\int_1^{\infty} \frac{1}{x^2+1}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^2+1}dx$

$\lim_{b\to\infty} \tan^{-1}(x)\Big|_1^b$

$\lim_{b\to\infty} \tan^{-1}(b) - \tan^{-1}(1)$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

∴ series converges

B. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$\int_1^{\infty} \frac{x}{x^2+1}dx = \lim_{b\to\infty} \int_1^b \frac{x}{x^2+1}dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}du$

$\lim_{b\to\infty} \frac{1}{2}\ln|x^2+1|\Big|_1^b$

$\lim_{b\to\infty} \frac{1}{2}(\ln|b^2+1| - \ln 2)$

$= \infty$

∴ series diverges

Comparing Two Series:
See proof of divergence of a harmonic series video on Weebly
(I like this proof more ☺)