

# Review of Sections 9.1 – 9.6 Sequences and Series

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Determine if the following sequences converge or diverge. If it converges, find its limit.

C to $1/2$	C to 0	C to 0
D		C to 1

Find the sum of the following geometric series

2	$9/13$	$\pi^2/(\pi-e)$
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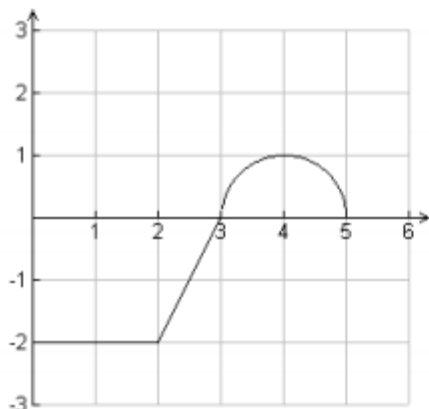
Determine if the following converge or diverge

Converge (Direct Comparison with $1/3^n$ )	Converges by Ratio Test	Converges by Limit Comparison
Diverges by alternating series test	Diverges by direct comparison with $1/n$	Converges by the Alternating Series Test
Converges by the root test	Converges by the Ratio Test	Diverges by the intergral test
Diverges by the root test	Converges by the Alternating Series Test	Converges by the Ratio Test
Converges by the Ratio Test	Diverges b/c $ r  > 1$	Diverges by the Alternating Series Test
Converges b/c $ r  < 1$	Converges p-series and $n > 1$	Diverges by the nth term test
Converges by the ratio test	Diverges by the nth term test	Converges b/c its telescopic or integral test
Converges by the ratio test	Converges by ratio test	Converges b/c it geometric and $ r  < 1$

Determine if the following diverges, converges absolutely, or converges conditionally

Converges conditionally	Converges absolutely	Diverges
Converges absolutley	Convereges absolutely	Converges absolutely

Free Response 1 – No Calculator



The graph of  $f$  is given. It consists of two line segments and a semi-circle.

$$g(x) = \int_1^x f(t) dt$$

- (a) Find  $g(0)$ ,  $g(1)$ , and  $g(5)$ .
- (b) Find  $g'(2)$ ,  $g''(2)$ , and  $g''(4)$  or state that it does not exist.
- (c) For what value(s) of  $x$  does the graph of  $g$  have a point of inflection? Justify your answer.
- (d) Find the absolute maximum and absolute minimum values of  $g$  on  $[0, 5]$ . Justify your answer.

(a)  $g(0) = \int_1^0 f(t) dt = 2$

$$g(1) = \int_1^1 f(t) dt = 0$$

$$g(5) = \int_1^5 f(t) dt = \frac{1}{2}\pi - 3$$

(b)  $g'(2) = f(2) = -2$

$$g''(2) = f'(2) = \text{DNE}$$

$$g''(4) = f'(4) = 0$$

- (c)  $g$  has a point of inflection at  $x = 4$  because  $g' = f$  changes from increasing to decreasing.

- (d) Candidates are  $x = 0, 3, 5$ , the endpoints of the interval and the critical number.

$x$	$g(x)$
0	2
3	-3
5	$\frac{1}{2}\pi - 3$

The absolute minimum value is  $-3$ .  
The absolute maximum value is 2.

2 pts: 1 pt  $g(0)$

1 pt  $g(1)$  and  $g(5)$

2 pts: 1 pt  $g''(2)$

1 pt  $g'(2)$  and  $g''(4)$

2 pts: 1 pt  $x = 4$

1 pt justification

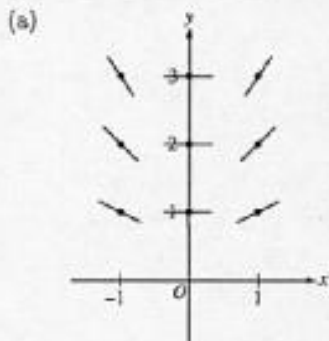
3 pts: 1 pt for candidates

1 pt evaluating candidates

1 pt for answers

4. Consider the differential equation given by  $\frac{dy}{dx} = \frac{xy}{2}$ .

- (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.
- (b) Let  $y = f(x)$  be the particular solution to the given differential equation with the initial condition  $f(0) = 3$ . Use Euler's method starting at  $x = 0$ , with a step size of 0.1, to approximate  $f(0.2)$ . Show the work that leads to your answer.
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 3$ . Use your solution to find  $f(0.2)$ .



1: line segments at nine points with negative - zero - positive slope left to right and increasing steepness bottom to top at  $x = 1$  and  $x = -1$

(b)  $f(0.1) \approx f(0) + f'(0)(0.1)$   
 $= 3 + \frac{1}{2}(0)(3)(0.1) = 3$   
 $f(0.2) \approx f(0.1) + f'(0.1)(0.1)$   
 $= 3 + \frac{1}{2}(0.1)(3)(0.1)$   
 $= 3 + \frac{.03}{2} = 3.015$

2 { 1: Euler's Method equations or equivalent table  
 1: answer (not eligible without first point)

Special Case: 1/2 for first iteration 3.015 and second iteration 3.045

(c)  $\frac{dy}{dx} = \frac{xy}{2}$   
 $\int \frac{dy}{y} = \int \frac{x}{2} dx$   
 $\ln |y| = \frac{1}{4}x^2 + C_1$   
 $y = Ce^{x^2/4}$   
 $3 = Ce^0 \implies C = 3$   
 $y = 3e^{x^2/4}$   
 $f(0.2) = 3e^{.04/4} = 3e^{.01} = 3.030$

6 { 1: separates variables  
 1: antiderivative of  $dy$  term  
 1: antiderivative of  $dx$  term  
 1: solves for  $y$   
 1: solves for constant of integration  
 1: evaluates  $f(0.2)$

Note: max 4/6 [1-1-1-0-0-1] if no constant of integration