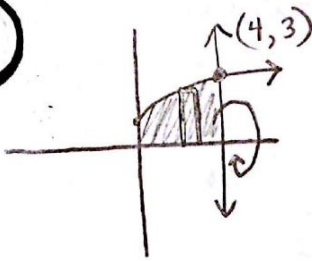


# Review: Area + Volume

①



$$y = \sqrt{x} + 1$$

$$\sqrt{4} + 1$$

$$y = 3$$

$$V = \pi \int_0^4 (\sqrt{x} + 1)^2 dx = \pi \int_0^4 (x + 2\sqrt{x} + 1) dx$$

$$(\sqrt{x} + 1)(\sqrt{x} + 1)$$

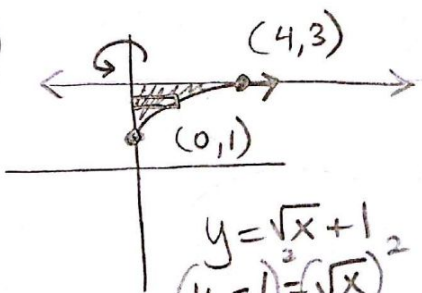
$$x + 2\sqrt{x} + 1$$

$$= \pi \left( \frac{x^2}{2} + 2 \cdot \frac{2}{3} x^{3/2} + x \Big|_0^4 \right)$$

$$= \pi \left( 8 + \frac{32}{3} + 4 \right)$$

$$= \frac{68\pi}{3}$$

②



$$y = \sqrt{x} + 1$$

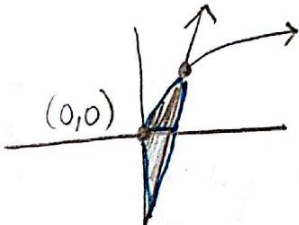
$$(y - 1)^2 = (\sqrt{x})^2$$

$$x = y^2 - 2y + 1$$

$$V = \pi \int_1^3 (y^2 - 2y + 1)^2 dy$$

$$= 20.106$$

③



$$(5\sqrt{x})^2 = (4x - 6)^2$$

$$25x = 16x^2 - 48x + 36$$

$$16x^2 - 73x + 36 = 0$$

$$(16x^2 - 64x) - 9x + 36$$

$$16x(x - 4) - 9(x - 4) = 0$$

$$16x - 9 = 0 \quad x - 4 = 0$$

$$x = \frac{9}{16} \quad x = 4$$

$$5\sqrt{4} = 4(4) - 6$$

$$10 = 10$$

$$5 \cdot \frac{3}{4} = 4 \left( \frac{9}{16} \right) - 6$$

$$\frac{15}{4} \neq \frac{-15}{4}$$

$$A = \int_0^4 (5\sqrt{x} - 4x + 6) dx$$

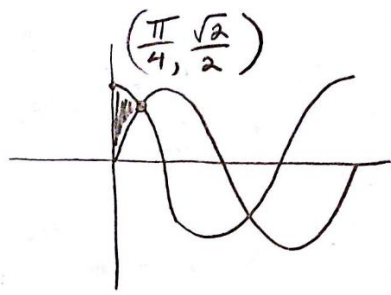
$$= 5 \cdot \frac{2}{3} x^{3/2} - \frac{4x^2}{2} + 6x \Big|_0^4$$

$$\frac{10}{3} x^{3/2} - 2x^2 + 6x \Big|_0^4$$

$$\frac{80}{3} - 32 + 24$$

$$= \frac{56}{3}$$

④



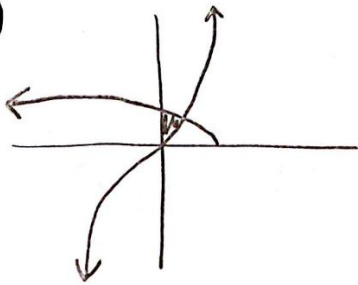
$$A = \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$A = -\sin x + \cos x \Big|_0^{\pi/4}$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2}$$

$$-(0 + 1) = \sqrt{2} - 1$$

⑤



$$A = \int_0^1 (\sqrt{2-x} - x^3) dx$$

$$= .970$$

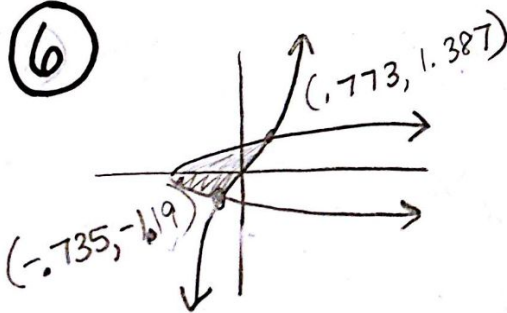
$$(x^3)^2 = (\sqrt{2-x})^2$$

$$x^6 = 2 - x$$

$$x^6 + x - 2 = 0$$

$$x = 1$$

⑥



$$A = \int_{-1.387}^{1.387} \left( \sqrt{\frac{x+5}{3}} - (3x^3)^2 \right) dy$$

$$= 8.68$$

$$\left( \sqrt{\frac{x+5}{3}} \right)^2 = (3x^3)^2$$

$$\frac{x+5}{3} = 9x^6$$

$$x+5 = 27x^6$$

$$27x^6 - x - 5 = 0$$

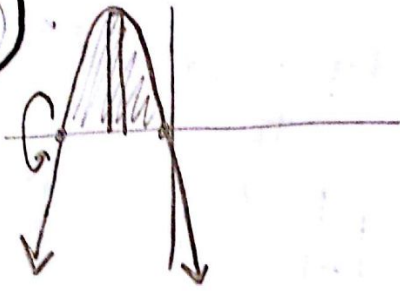
$$\pm \sqrt{\frac{x+5}{3}} = y$$

$$y = 3x^3$$

$$\frac{y}{3} = x^3$$

$$x = \sqrt[3]{\frac{y}{3}}$$

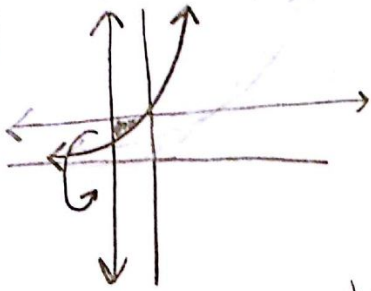
7



$$V = \pi \int_{-5}^3 (15 - 2x - x^2)^2 dx$$

$$= 3431.460$$

8



$$V = \pi \int_{-1}^0 1 - (e^x)^2 dx$$

$$V = \pi \left( x - \frac{1}{2} e^{2x} \Big|_{-1}^0 \right)$$

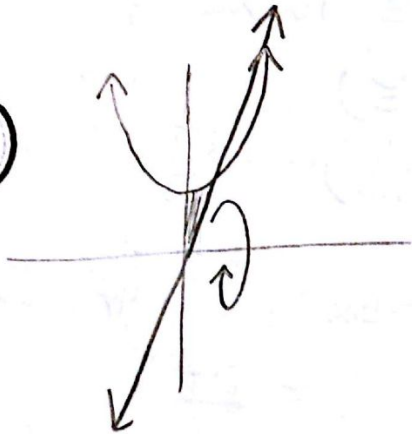
$$V = \pi \left( -\frac{1}{2} - \left( -1 - \frac{1}{2} e^{-2} \right) \right)$$

$$\pi \left( -\frac{1}{2} + 1 + \frac{1}{2} e^{-2} \right)$$

$$\pi \left( \frac{1}{2} + \frac{1}{2} e^{-2} \right)$$

$$\frac{1}{2} \pi (1 + e^{-2})$$

9



$$x^2 + 6 = 5x$$

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$$x = 2, 3$$

$$V = \pi \int_2^3 (x^2 + 6)^2 - (5x)^2 dx$$

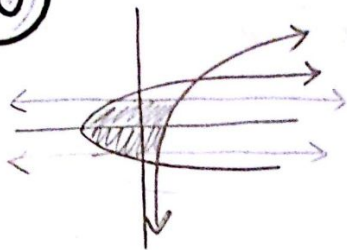
$$\pi \int_2^3 (x^4 + 12x^2 + 36 - 25x^2) dx$$

$$\pi \int_2^3 (x^4 - 13x^2 + 36) dx$$

$$\pi \left( \frac{x^5}{5} - \frac{13x^3}{3} + 36x \Big|_2^3 \right)$$

$$\pi \left( \frac{324}{5} - \frac{104}{3} + 72 \right) = \frac{656}{15} \pi$$

10



$x = e^y$   
 $y = \ln x$   
 $x = y^2 - 2$   
 $\sqrt{x+2} = \sqrt{y^2}$   
 $y = \sqrt{x+2}$

$$A = \int_{-1}^1 (e^y - y^2 + 2) dy$$

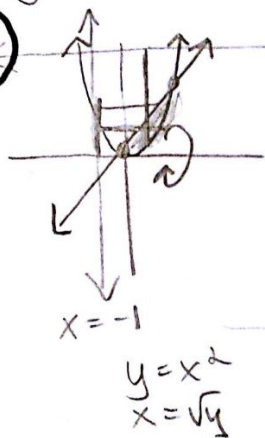
$$= e^y - \frac{y^3}{3} + 2y \Big|_{-1}^1$$

$$= (e - \frac{1}{3} + 2) - (e^{-1} + \frac{1}{3} - 2)$$

$$e + \frac{5}{3} - \frac{1}{e} - \frac{1}{3} + \frac{6}{3}$$

$$e - \frac{1}{e} + \frac{10}{3}$$

11



a) x-axis

$$V = \pi \int_0^1 [x^2 - (x^2)^2] dx = \pi \int_0^1 (x^2 - x^4) dx$$

$$= \pi \left( \frac{x^3}{3} - \frac{x^5}{5} \Big|_0^1 \right)$$

$$= \pi \left( \frac{1}{3} - \frac{1}{5} \right)$$

$$= \pi \left( \frac{5}{15} - \frac{3}{15} \right) = \frac{2\pi}{15}$$

b) y=2

$$V = \pi \int_0^1 [(2-x^2)^2 - (2-x)^2] dx = \pi \int_0^1 (-5x^2 + x^4 + 4x) dx$$

$$= \frac{8\pi}{15}$$

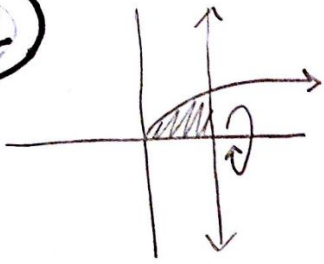
$$\frac{(2-x^2)(2-x^2)}{4 - 4x^2 + x^4 - (4 - 4x + x^2)} = \pi \int_0^1 (-y^2 + 2y^{1/2} - y) dy$$

c)  $V = \pi \int_0^1 (\sqrt{y}+1)^2 - (y+1)^2 dy$

$$= \pi \int_0^1 (y + 2\sqrt{y} + 1 - (y^2 + 2y + 1)) dy = \pi \left( -\frac{y^3}{3} + 2 \cdot \frac{2}{3} y^{3/2} - \frac{y^2}{2} \Big|_0^1 \right)$$

$$= \pi \left( -\frac{1}{3} + \frac{4}{3} - \frac{1}{2} \right) = \frac{\pi}{2}$$

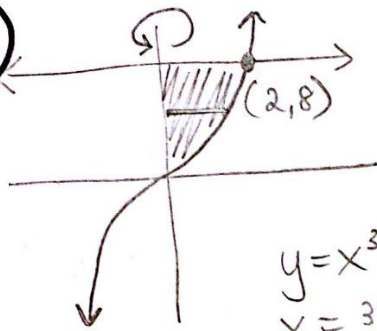
12



$$V = \pi \int_0^1 (\sqrt{x})^2 dx = \pi \int_0^1 x dx$$

$$= \pi \frac{x^2}{2} \Big|_0^1 = \frac{\pi}{2}$$

13



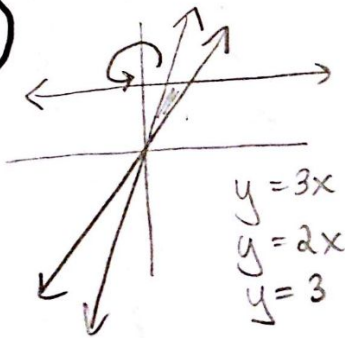
$$y = x^3$$

$$x = \sqrt[3]{y} = y^{1/3}$$

$$V = \pi \int_0^8 (y^{1/3})^2 dy = \pi \frac{3}{5} y^{5/3} \Big|_0^8$$

$$\frac{96\pi}{5}$$

14



$$y = 3x$$

$$y = 2x$$

$$y = 3$$

$$x = \frac{y}{3}$$

$$x = \frac{y}{2}$$

$$V = \pi \int_0^3 \left( \frac{y}{2} \right)^2 - \left( \frac{y}{3} \right)^2 dy$$

$$V = \pi \int_0^3 \left( \frac{y^2}{4} - \frac{y^2}{9} \right) dy$$

$$V = \pi \left( \frac{1}{4} \cdot \frac{y^3}{3} - \frac{1}{9} \cdot \frac{y^3}{3} \Big|_0^3 \right)$$

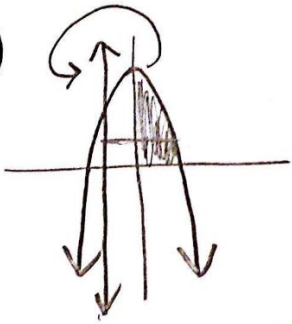
$$V = \pi \left( \frac{y^3}{12} - \frac{y^3}{27} \Big|_0^3 \right)$$

$$V = \pi \left( \frac{27}{12} - 1 \right)$$

$$V = \pi \left( \frac{15}{12} \right)$$

$$= \frac{5\pi}{4}$$

15



$$y = 9 - x^2$$

$$y - 9 = -x^2$$

$$\sqrt{-y + 9} = \sqrt{x^2}$$

$$x = \sqrt{9 - y}$$

$$V = \pi \int_0^9 ((\sqrt{9-y} + 2)^2 - 2^2) dy$$

$$V = \pi \int_0^9 (9 - y + 4(9 - y)^{1/2}) dy$$

$$(\sqrt{9-y} + 2)(\sqrt{9-y} + 2)$$

$$9 - y + 4\sqrt{9-y} + 4$$

$$13 - y + 4(\sqrt{9-y}) - 4$$

$$9 - y + 4\sqrt{9-y}$$

$$u = 9 - y$$

$$du = -dy$$

$$-du = dy$$

$$-4 \int u^{1/2} du$$

$$-4 \cdot \frac{2}{3} u^{3/2}$$

$$-\frac{8}{3} u^{3/2}$$

$$-\frac{8}{3} (9 - y)^{3/2}$$

$$\pi \left( 9y - \frac{y^2}{2} + \frac{8}{3} (9 - y)^{3/2} \Big|_0^9 \right)$$

$$\pi \left[ \left( 81 - \frac{81}{2} + 0 \right) - \left( 0 - 72 \right) \right]$$

$$\pi \left( \frac{225}{2} \right)$$