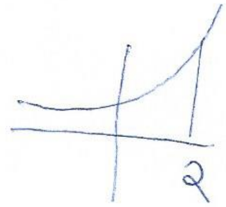
Review for Differential Equations

The graph of $y = 5x^4 - x^5$ has a point of inflection at

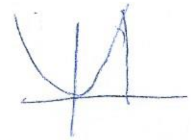
(A) (0,0) only (B) (3,162) only (C) (4,256) only

(D) (0,0) and (3,162) (E) (0,0) and (4,256)

The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is



(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$



The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

$\int_0^4 2^2 - (\sqrt{x})^2$

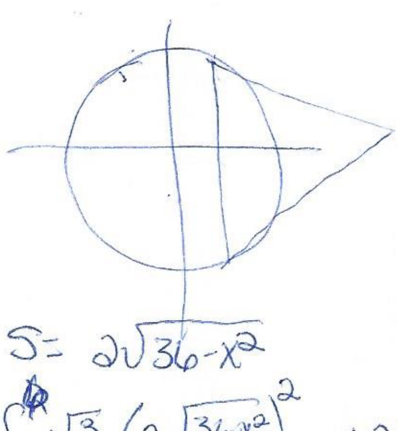
Solve the differential equation $e^{x+y}\,dy = dx$.

a) $e^{-x} + e^{y} = C$ b) $e^{-x} + e^{-y} = C$ c) $e^{x} + e^{y} = C$

d) $e^{x} + e^{-y} = C$ e) $e^{-x} + 2e^{-y} = C$

The base of a solid is a circular region in the xy-plane bounded by the graph of $x^2 + y^2 = 36$. Find the volume of the solid if every cross section by a plane normal to the x-axis is an equilateral triangle with one side on the base.

a) $72\sqrt{3}$ b) $288\sqrt{3}\pi$ c) $144\sqrt{3}\pi$ d) $144\sqrt{3}$ e) $288\sqrt{3}$



$e^y e^x\,dy = dx$

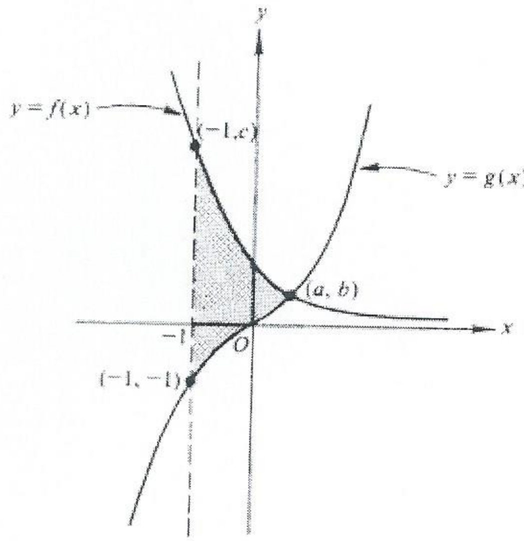
$e^y\,dy = e^{-x}\,dx$

$e^y = -e^{-x} + C$

$e^y + e^{-x} = C$

$S = 2\sqrt{36-x^2}$

$\int_{-6}^{6} \frac{\sqrt{3}}{4}\left(2\sqrt{36-x^2}\right)^2 = 2\sqrt{3}\int_0^6 36 - x^2\,dx$



The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a (f(x) - g(x))\,dx + \int_{-1}^0 (f(x) + g(x))\,dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c (f(x) - g(x))\,dx$

(D) $\int_{-1}^a (f(x) - g(x))\,dx$

(E) $\int_{-1}^a (|f(x)| - |g(x)|)\,dx$

Which of the following gives the area of the surface generated by revolving about the y-axis the arc of $x = y^3$ from $y = 0$ to $y = 1$?

?

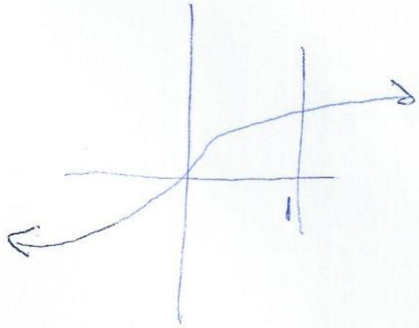
(A) $2\pi \int_0^1 y^3 \sqrt{1 + 9y^4}\,dy$

(B) $2\pi \int_0^1 y^3 \sqrt{1 + y^6}\,dy$

(C) $2\pi \int_0^1 y^3 \sqrt{1 + 3y^2}\,dy$

(D) $2\pi \int_0^1 y \sqrt{1 + 9y^4}\,dy$

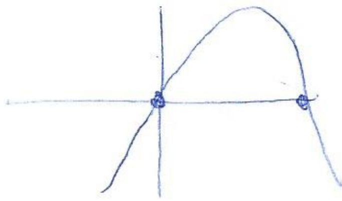
(E) $2\pi \int_0^1 y \sqrt{1 + y^6}\,dy$



$x(6-x)$

The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

Shell Method
Omit



(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

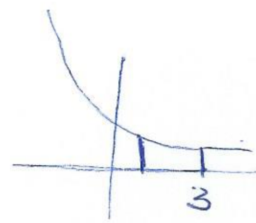
(B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$

(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9-y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3 + \sqrt{9-y}\right)^2 dy$

The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is



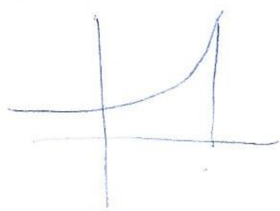
(A) $\dfrac{\left(1 - e^{-6}\right)}{2}$ (B) $\dfrac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1 - e^{-3}$

$S = e^{-x}$

$\int_0^3 \left(e^{-x}\right)^2$

A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

Shell Method
Omit



(A) $2\pi\int_0^1 xe^{2x}\,dx$

(B) $2\pi\int_0^1 e^{2x}\,dx$

(C) $\pi\int_0^1 e^{4x}\,dx$

(D) $\pi\int_0^e y\ln y\,dy$

(E) $\frac{\pi}{4}\int_0^e \ln^2 y\,dy$

An equation of the line tangent to the graph of $y = \dfrac{2x+3}{3x-2}$ at the point $(1, 5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

$y - 5 = -13(x-1)$

$y - 5 = -13x + 13$

If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

$\pi\int_0^2 y^2\,dy$

An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x+1$ (B) $y = x+1$ (C) $y = x$ (D) $y = x-1$ (E) $y = 0$

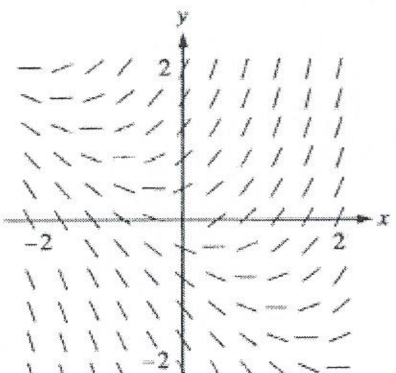
$1 - \sin t$

$y - 1 = (t - 0)$

What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

$\int_0^2 x^2 - (-x)$



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

The area enclosed by the curves $y = x^2 - 4$ and $y = 2x - 4$ can be represented by the integral

(A) $\int_{-4}^{0} (x^2 - 2x)\,dx$

(B) $\int_{0}^{2} (2x - x^2)\,dx$

(C) $\int_{-4}^{0} (2x - x^2)\,dx$

(D) $\int_{0}^{2} (x^2 - 2x)\,dx$

(E) $\int_{-4}^{2} (x^2 - 2x)\,dx$

$2x - 4 - (x^2 - 4)$

1. Find the area under the graph of $y = x + 2$ on the interval $[1, 3]$.
(A) 2
(B) 3
(C) 5
(D) 8
(E) 15

2. Find the area between the x-axis and the graph of $y = |x|$ on the interval $[-10, 10]$.
(A) 0
(B) 10
(C) 20
(D) 50
(E) 100

3. Find the area between the x-axis and the graph of $y = 3 - |x|$ on the interval $[0, 6]$.
(A) 0
(B) $\frac{9}{2}$
(C) 9
(D) 18
(E) 36

4. Find the area bounded by the graph of $f(x) = \sqrt{16 - x^2}$ and the x-axis.
(A) π
(B) 2π
(C) 4π
(D) 8π
(E) 16π

7. Find the area bounded by the graphs of $y = x^2 - 4x$ and $y = x$.
(A) $\frac{25}{3}$
(B) 10
(C) 12.5
(D) $\frac{125}{6}$
(E) 21

8. The area enclosed by the graphs of $y = e^x$, $y = x$, the y-axis, and the line $x = 2$ is equal to
(A) e^2
(B) $e^2 - 1$
(C) $e^2 + 1$
(D) $e^2 - 3$
(E) $e^2 - 2$

1. The base of a solid is the region enclosed by $y = \sin x$ and the x-axis on the interval $[0, \pi]$. Cross sections perpendicular to the x-axis are semicircles with diameter in the plane of the base. Write an integral that represents the volume of the solid.
(A) $\frac{\pi}{8}\int_0^{\pi} (\sin x)^2\,dx$
(B) $\frac{\pi}{8}\int_0^{1} (\sin x)^2\,dx$
(C) $\frac{\pi}{4}\int_0^{\pi} (\sin x)^2\,dx$
(D) $\frac{\pi}{8}\int_0^{\pi} \sin x\,dx$
(E) $\frac{\pi}{2}\int_0^{\pi} (\sin x)^2\,dx$

2. The base of a solid is the region enclosed by $y = e^x$, the x-axis, the y-axis, and the line $x = \ln 3$. Cross sections perpendicular to the x-axis are squares. Write an integral that represents the volume of the solid.
(A) $\int_0^{\ln 3} e^x\,dx$
(B) $\int_0^{(\ln 3)^2} e^{2x}\,dx$
(C) $\int_0^{\ln 3} e^{2x}\,dx$
(D) $\pi\int_0^{\ln 3} e^{2x}\,dx$
(E) $\pi\int_0^{3} e^{2x}\,dx$

5. Find the volume of the solid formed by rotating about the x-axis the region enclosed by the graph of $y = \sqrt{x} + 1$, the x-axis, the y-axis, and the line $x = 4$.
(A) 7.667
(B) 9.333
(C) 22.667
(D) 37.699
(E) 71.209

6. Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis, and the line $y = 3$ about the y-axis.
(A) 6.40
(B) 8.378
(C) 20.106
(D) 100.531
(E) 145.77

7. Find the volume of the solid formed by rotating the region bounded by the graph of $y = \sqrt{x} + 1$, the y-axis, and the line $y = 3$ about the line $y = 5$.
(A) 13.333
(B) 17.657
(C) 41.888
(D) 92.153
(E) 242.95

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Which of the following integrals represents the volume of the solid obtained by rotating the region bounded by the graph of $y = -\sqrt{x}$, the x-axis, and the line $x = 4$ about the y-axis?

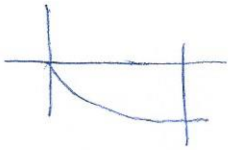
(A) $\pi \int_0^4 y^2\,dy$

(B) $\pi \int_0^2 y^2\,dy$

(C) $\pi \int_0^4 (\sqrt{x})^2\,dx$

(D) $\pi \int_0^2 (-\sqrt{x})^2\,dx$

(E) $\pi \int_0^4 (-x)\,dx$



$x = y^2$

9

The differential equation $\frac{dy}{dx} = xy + 2y$ in separable form is

$y(x+2)$

(A) $\frac{dy}{xy + 2y} = dx$

(B) $dy = (xy + 2y)\,dx$

(C) $\frac{dy}{dx} - xy = 2y$

(D) $dy = (x + 2)y\,dx$

(E) $\frac{dy}{y} = (x + 2)\,dx$

4. A possible solution to $\frac{dy}{dt} = y - 5$ is

(A) $y = 5 + Ce^t$

(B) $y = Ce^t - 5$

(C) $y = 5e^t$

(D) $y^2 - 5y = t + c$

(E) $y = Ce^{-5t}$

$\frac{dy}{y-5} = dt$

$\ln(y-5) = t + C$

$e^{t+C} = y - 5$

5. The solution to the differential equation $\frac{dP}{dt} = -0.02P$ with $P(0) = 5$ is

(A) $P = 5e^{-0.02t}$

(B) $P = 5e^{-0.002t}$

(C) $P = 5 + e^{-0.02t}$

(D) $P = -0.02P^2 + c$

(E) $P = \frac{1}{5 - 0.02t}$

$y = e^{kt}$

1. The differential equation $\frac{dy}{dx} = 2y + 50$ written in separable form is

(A) $\frac{1}{2y}\frac{dy}{dx} = 50$

(B) $\frac{dy}{y + 50} = 2dx$

(C) $dy = (2y + 50)dx$

(D) $\frac{dy}{y + 25} = 2dx$

(E) $\frac{dy}{2y} = 50dx$

2. If $y = \ln(e^{-t^2} + 10)$, then $\frac{dy}{dx} =$

(A) $-2t$

(B) $\frac{1}{e^{-t^2} + 10}$

(C) $\frac{-2te^{-t^2}}{e^{-t^2} + 10}$

(D) $\frac{-2t}{e^{-t^2} + 10}$

(E) $-2t + \frac{1}{10}$

$\frac{-2t\,e^{-t^2}}{e^{-t^2} + 10}$

2. The x-coordinate(s) of the point(s) of inflection of $f(x) = \frac{x}{x^2 + 1}$ is (are)

(A) 0

(B) ± 1

(C) $\pm\sqrt{3}$

(D) 0 and $\pm\sqrt{3}$

(E) no points of inflection

$\frac{x^2 + 1 - x(2x)}{(x^2+1)^2}$ $\frac{1 - x^2}{(x^2+1)^2}$

7. Find $\frac{dy}{dx}$ if $y = 3x(x - 2)^3$.

(A) $9(x - 2)^2$

(B) $9x(x - 2)^2$

(C) $(12x - 6)(x - 2)^2$

(D) $3(x - 2)^3$

(E) $(2x - 1)(x - 2)^2$

$3[(x-2)^3 + 3x(x-2)^2]$

$3(x-2)^2[x - 2 + 3x]$

$3(x-2)^2(4x-2)$

1. Find the equation of the line tangent to $f(x) = 2x + 2e^x$ at $x = 0$.

(A) $y = 4x + 2$

(B) $y = 2x + 2$

(C) $y = 4x$

(D) $y = 4x - 2$

(E) $y = -\frac{1}{4}x + 2$

$y - 2 = 4(x - 0)$

$y'\ 2 + 2e^x$

$y'(0) = 4$

$y(0) = 2$