Day 3 – More related rates

Objective: Use basic math formulas and derivatives to solve problems

1. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep.

$r=2$, $h=4$, $r=\frac{1}{2}h$

$\frac{dV}{dt}=2\ m^3/min$, $\frac{dh}{dt}=?$ when $h=3$

$\boxed{V=\frac{1}{3}\pi r^2 h}$

$V=\frac{1}{3}\pi\left(\frac{1}{2}h\right)^2\cdot h$

$V=\frac{1}{3}\pi\left(\frac{1}{4}h^2\right)\cdot h$

$V=\frac{1}{12}\pi h^3$

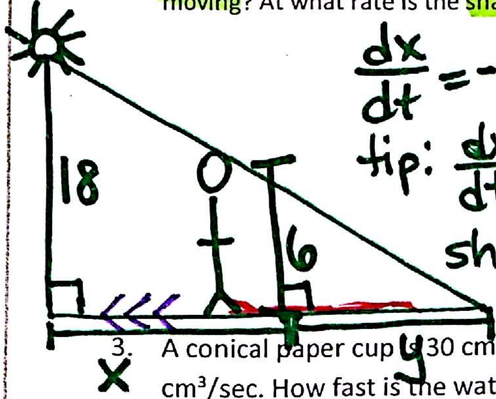
$\frac{dV}{dt}=\frac{1}{4}\pi h^2\frac{dh}{dt}$

$2=\frac{1}{4}\pi(3)^2\left(\frac{dh}{dt}\right)$

$8=9\pi\frac{dh}{dt}$

$\frac{8}{9\pi}=dh/dt$ m/min

2. A man 6ft tall man is walking at the rate of 3 ft/s toward a streetlight 18 ft high. At what rate is the tip of the shadow moving? At what rate is the shadow length changing?



$\frac{dx}{dt}=-3\ ft/s$

tip: $\frac{dx}{dt}+\frac{dy}{dt}$?

shadow: $\frac{dy}{dt}$?

$\frac{x+y}{18}=\frac{y}{6}$

$6x+6y=18y$

$x+y=3y$

$x=2y$

$\frac{dx}{dt}=2\frac{dy}{dt}$

$-3=2\frac{dy}{dt}$

$\frac{dy}{dt}=-\frac{3}{2}$ ft/s SHADOW

$-\frac{3}{2}+-3=\frac{-9}{2}$ ft/s TIP

3. A conical paper cup is 30 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of 2π 3 cm³/sec. How fast is the water level rising when the water level is 2 cm?

$\frac{2\pi}{3}$

$h=30$, $r=10$, $r=\frac{1}{3}h$

$\frac{dV}{dt}=\frac{2\pi}{3}$, $\frac{dh}{dt}=?$ when $h=2$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2\cdot h$

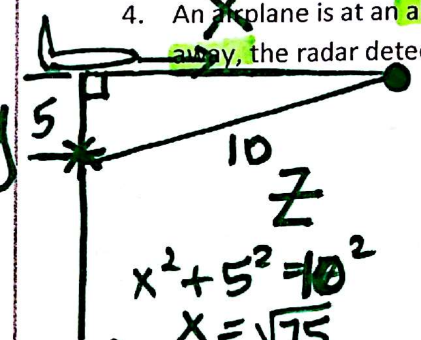
$V=\frac{1}{27}\pi h^3$

$\frac{dV}{dt}=\frac{1}{9}\pi h^2\frac{dh}{dt}$

$\frac{2\pi}{3}=\frac{1}{9}\pi(2)^2\cdot\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{18\pi}{12\pi}=\frac{3}{2}$ cm/sec

4. An airplane is at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance s is changing at a rate of 240 mph. What is the speed of the airplane?



$x^2+5^2=10^2$

$x=\sqrt{75}$

$\frac{dz}{dt}=240\ mi/hr$

$y=5$ (constant)

$z=10$

$\frac{dx}{dt}=?$

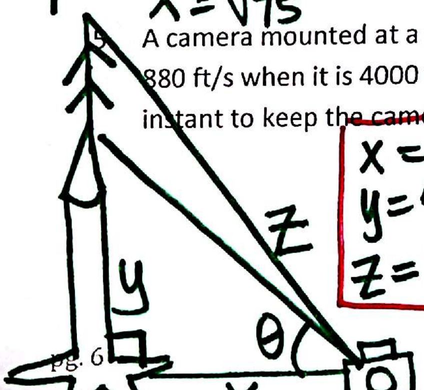
$x^2+y^2=z^2$

$2x\frac{dx}{dt}+\cancel{2y\frac{dy}{dt}}=2z\frac{dz}{dt}$

$2\sqrt{75}\frac{dx}{dt}+0=2(10)(240)$

$\frac{dx}{dt}=\frac{2400}{\sqrt{75}}$ mi/hr

5. A camera mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket?



$\boxed{\begin{aligned}x&=3000\\y&=4000\\z&=5000\end{aligned}}$

$\frac{dy}{dt}=880$ ft/s

$\frac{d\theta}{dt}=?$

$\tan^{-1}\tan\theta=\tan^{-1}\left(\frac{4000}{3000}\right)$

$\theta=.927$

$\tan\theta=\frac{y}{x}$

$\tan\theta=\frac{1}{3000}y$

$\sec^2\theta\frac{d\theta}{dt}=\frac{1}{3000}\frac{dy}{dt}$

$\sec^2(.927)\left(\frac{d\theta}{dt}\right)=\frac{1}{3000}(880)$

.106 rad/s