Day 2 – Related Rates

Objective: Use related rates to solve problems

Example #1

A particle moves along the curve $\boxed{y = 3x^2 - 6x}$ so that the rate of change of the x-coordinate dx/dt is 2 units/sec. Find the rate of change of the y-coordinate, dy/dt, when the particle is at the origin. $(0,0)$

Take the derivative of both sides with respect to t	$\frac{dy}{dt} = 6x\frac{dx}{dt} - 6\frac{dx}{dt}$
Substitute in the known info	$\frac{dy}{dt} = 6(0)(2) - 6(2)$
Solve for the indicated variable	$\frac{dy}{dt} = -12$ units/sec

A strategy for Solving Related Rates Problems

Step 1	Read the problem carefully.
Step 2	Draw a diagram if possible
Step 3	Introduce notation. Assign symbols to all quantities that are functions of time
Step 4	Express the given information and the required rate in terms of derivatives
Step 5	Write an equation that relates the various quantities of the problem. If necessary, use Geometry to eliminate one of the variables by substitution
Step 6	Use the Chain Rule to differentiate both sides of the equation with respect to t
Step 7	Substitute the given information in the resulting equation and solve for the unknown rate.

Example 2:

Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm³/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm? The volume of a sphere is $V = \frac{4}{3}\pi r^3$. (You will not be given this formula on the quiz or test)

$\frac{dV}{dt} = 100\ cm^3/s$

$\frac{dr}{dt} = ?$

when $d = 50$

$r = 25$

$V = \frac{4}{3}\pi r^{3}$

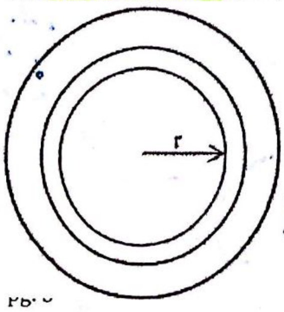
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{100}{4\pi(25)^2} = \frac{4\pi(25)^2 \cdot \frac{dr}{dt}}{4\pi(25)^2}$

$\frac{dr}{dt} = \frac{1}{25\pi}$ cm/sec

Example 3:

Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2ft/sec. How fast is the area of the spill increasing when the radius of the spill if 60ft?



$\frac{dr}{dt} = 2$ ft/sec

$\frac{dA}{dt} = ?$

when $r = 60$

$A = \pi r^2$

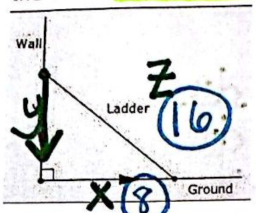
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(60)(2)$

$\frac{dA}{dt} = 240\pi\ ft^2/sec$

Example 4

A 16-foot ladder leans against a wall. The bottom of the ladder is 5ft from the wall at time t = 0 and slides away from the wall at a rate of 3 ft/ sec. Find the velocity of the top of the ladder at time t = 1.



$\frac{dx}{dt} = 3$ ft/s

$\frac{dy}{dt} = ?$

$z = 16$

$t=0,\ x=5$

$t=1,\ \underline{x=8}$

$8^2 + y^2 = 16^2$

$y = \sqrt{192}$

$x^2 + y^2 = z^2$

$x^2 + y^2 = 16^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

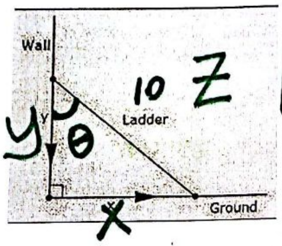
$2(8)(3) + 2(\sqrt{192})\left(\frac{dy}{dt}\right) = 0$

$2\sqrt{192}\,\frac{dy}{dt} = -48$

$\frac{dy}{dt} = \frac{-24}{\sqrt{192}}$ ft/sec

Example 5

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is π/4 radian?



$z = 10$

$\left[\frac{dz}{dt} = 0 \text{ ft/s}\right]$

$\frac{dx}{dt} = 2$ ft/s

$\frac{d\theta}{dt} = ?$

when $\theta = \frac{\pi}{4}$

$\sin\theta = \frac{x}{10}$

$\sin\theta = \frac{1}{10}x$

$\cos\theta\,\frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{dx}{dt}$

$\cos\left(\frac{\pi}{4}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{10}\cdot 2$

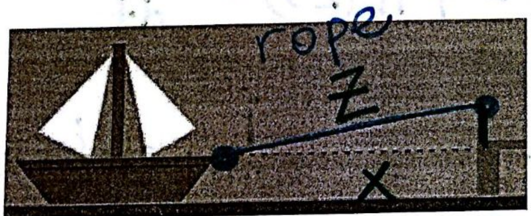
$\frac{\sqrt{2}}{2}\cdot\frac{d\theta}{dt} = \frac{1}{5}$

$\frac{d\theta}{dt} = \frac{2}{5\sqrt{2}}$ rad/sec

$\frac{\sqrt{2}}{5}$

Example #6

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 meter higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?



$-\frac{\sqrt{65}}{8}$ m/sec

$x^2 + y^2 = z^2$

$x^2 + 1 = z^2$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$2(8)\left(\frac{dx}{dt}\right) = 2(\sqrt{65})(-1)$